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Health Expectancy Calculation by the Sullivan Method: A Practical Guide

4th Edition

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CONTENTS

Tables

Foreword

This guide was originally developed in the Euro-REVES project and has become a well-cited reference for calculations. With the development of the European Health Expectancy Monitoring Unit (EHEMU) (2005-7) and renewed efforts on data harmonisation and training, it was felt that the guide would benefit from updating of techniques and more recent data in the examples. Further small amendments were made during the European Health and Life Expectancy Information System (EHLEIS) project and this guide together with a shorter non-technical guide to understanding health expectancies was made available on the EHLEIS website (www.eurOhex.eu). In 2014 funding was provided by the Japan Society for the Promotion of Science (Grant-in-Aid for Scientific Research 25293121) to enable the addition of a new section to show the standard error of the proportion of healthy life.

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Summary

What is Health Expectancy?

Life expectancy is composed of lengths of time spent in different states of health until death. These lengths of time in different states of health are health expectancies and they combine information on both mortality and morbidity.

What is Health Expectancy by Sullivan's method?

The concept of health expectancies as health indicators was proposed in 1964 (Sanders 1964) and the first example was published in a report of the US Department of Health Education and Welfare (Sullivan 1971). This report contained preliminary estimates of "Disability-Free Life Expectancy" calculated using a method devised by Sullivan and applicable to any state of health definition. We shall use the term Sullivan Health Expectancy as shorthand for health expectancy calculated by the Sullivan method.

The Sullivan health expectancy reflects the current health of a real population adjusted for mortality levels and independent of age structure. Health expectancy calculated by Sullivan's method is the number of remaining years, at a particular age, which an individual can expect to live in a healthy state (however health may be defined). For example, in 2004 in Belgium, women at age 65 could expect to live a further 20.0 years of which 12.4 years (62%) would be spent without disability, disability being defined as restrictions in daily activities due to longstanding illness(es), condition(s) or handicap(s) (see Example 1 Table 3.3).

What information is required to calculate the Sullivan health expectancy?

The data required are the age-specific prevalence (proportions) of the population in healthy and unhealthy states (often obtained from cross-sectional surveys), and age-specific mortality information taken from a period life table. Sullivan health expectancy is not very sensitive to the size of the age groups, meaning that an abridged life table[1] may be used. Generally, it is preferable to use five- (sometimes ten-) year age intervals because most surveys used to derive the age-specific proportions (prevalence) in healthy and unhealthy states are too small to allow smaller age intervals. This is particularly important at the higher ages and it is usual to include the final age interval as 85 years and over.

What is the Sullivan health expectancy used for?

Sullivan health expectancy provides a means of comparing the health states of an entire population at two time points or of two different populations at the same time point, despite any differences in age composition (provided that the age intervals are not too large). The same definitions of health states and age intervals must be used for the populations and/or time points being compared. Comparability is also increased by the calculation of this indicator separately for males and females and if necessary (to adjust for structural differences) for other categories.

If comparisons are to be made between several health expectancies, a number of provisos need to be made.

[1] An abridged life table is like a complete life table but has age intervals greater than one year (apart from the first years of life). A typical set of intervals might be 0 to under 1 year, 1 to under 5 years, 5 to under 10 years, 10 to under 15 years, etc.

- It is important that the same definitions of health states are used for each of the health expectancies. For example, differences between health expectancies calculated for different countries can often be explained by differences in the measurement instruments used to collect the prevalence data (Boshuizen and Van de Water 1994).
- The general frameworks of the surveys used to derive prevalence also need to be the same to allow comparisons. The estimates of the prevalence of ill-health are very sensitive to the way the data are collected (e.g. face-to-face interview, telephone interview, postal questionnaire).
- Some surveys exclude institutionalized persons, which may result in bias particularly for older populations and certain health conditions (Ritchie, Jagger et al. 1993). It may be possible to incorporate data from separate surveys of the institutionalized population although this often requires strong assumptions to be made (see Example 5).

Formal statistical comparisons may be made between times, places or subgroups, as the standard errors of estimates are relatively simple to calculate. Sullivan health expectancies have provided results for populations of over 50 countries worldwide (Jagger and Robine 2011) and are annually reported for all European Union countries (see Country Reports available at www.eurOhex.eu). The increasing interest in health expectancy as a policy tool is shown by its inclusion in a wide range of EU policy documents (Jagger, McKee et al. 2013).

What are the differences between the Sullivan method and other methods to calculate health expectancy?

To calculate health expectancy at a particular age and time, it is necessary to calculate the number of person years lived in the health state from that age at the particular time. Thus, theoretically, estimates of health expectancies at this time depend on the incidence of health states and are essentially longitudinal measures. Direct calculation of the person years lived in the health state requires longitudinal data to provide the transition rates between health states (multistate method). The Sullivan method is of particular practical importance as it uses more readily available data, age-specific prevalence of the health state and the total person years lived at a particular age. Obviously there must be some error associated with this approximation (except if all population characteristics are stable in time) but experience has shown that the Sullivan method can, generally, be recommended for its simplicity, relative accuracy and ease of interpretation. More detail on multistate methods for longitudinal data, including the different software options, have been documented(Robine, Jagger et al. 2003; Jagger and Robine 2011; Saito, Robine et al. 2014; Willekens and Putt 2014).

In particular, the Sullivan and multistate methods seem to produce similar results providing all transition rates are smooth and regular over time (Robine and Mathers 1993). The Sullivan method may underestimate (or overestimate) health expectancy when prevalence remains the same between two periods, whereas incidence rates between states of health change rapidly, because the prevalence of ill health at a given age in the population reflects the past probabilities of becoming ill at each younger age (Mathers 1991). In this case though, the Sullivan health expectancy remains a meaningful indicator of the state of health of a population, rather than prediction at an individual level. Differences between the multistate and Sullivan indicators also occur if prevalence changes (e.g. because of a decrease in mortality) whilst incidence remains constant (Barendregt, Bonneux et al. 1994). Many researchers have commented on the differences between the Sullivan and multistate methods (Rogers, Rogers et al. 1990; Bebbington 1991; Mathers 1991; Robine and Ritchie 1991; van de Water, Boshuizen et al. 1995).

The remainder of this book provides easy-to-follow examples of health expectancy calculations using the Sullivan method. Readers who require more technical explanations of the issues underlying these calculations will find references at the relevant points to sections in Appendix 1. Life tables are available in most countries from birth to age 90 years or beyond by single years of age (unabridged life tables). However, in contrast, the age-specific prevalence of the population in healthy (or unhealthy) state is rarely available on the full age range (since surveys often include only the adult population say age 16 years and above, or the older population aged 50 years and above) and is usually too small to tabulate by single years of age. We can match up the life table data and the prevalence data in two main ways. Either we can make assume that the prevalence of the health state for a given age group is the same for all the single years of age within that age group using an unabridged life table (Example 1) or we can make an abridged life table out of the unabridged one to match up with the age groups of the prevalence data (Example 2). For some countries the death counts for the life table may not be available by single year of age and so in Example 3 we show how to calculate an abridged life table and use this for the health expectancy. The fourth example documents the calculation of the standard error and confidence interval for the health expectancy, whilst the fifth example tests the equality of two health expectancies. The sixth example demonstrates how to calculate health expectancy when the age-specific prevalence of the health states are obtained from community living and institutionalized persons separately. Finally the last example shows how to calculate the standard error of another often reported quantity, health expectancy as a percentage of life expectancy at a particular age. These examples are on a Microsoft Excel spreadsheet available from the EHLEIS website (http://www.eurOhex.eu) where there are also SAS and SPSS programs of the examples.

Sullivan health expectancy using an unabridged life table: Example 1

We will show the calculation for disability-free life expectancy for women in Belgium in 2004. The calculations are presented in tabular form in Tables 1.2-1.5 and are easily performed on a spreadsheet (e.g. Microsoft Excel, QuattroPro or Lotus 1-2-3). Recall that the data required for health expectancy calculations are morbidity data (in the form of age-specific prevalence) and mortality data (from a period life table).

Morbidity data

The prevalence of disability by sex and age group are taken from the Belgian Health Interview Survey conducted in 2004. The survey does not include people living in institutions (in Example 6 we show how to combine data from a community survey with data on institutions) and has a complex sampling design. To adjust for the design characteristics of the survey, weighted prevalence rates were used. Further details of the survey are available (Bayingana, Demarest et al. 2006). The disability prevalence rates were obtained by means of the following question: "*Are you restricted in daily activities as a result of longstanding illness(es), condition(s) or handicap(s)?*". The answer "*all the time*" was defined as severe disability, "*now and then*" as moderate disability, and "*seldom*" or "*no*" as no disability. Table 1.1 shows the prevalence rates in 5 year age bands (1-4 years, 5-9 years, to age 85+ years). For the 65-69 year age group in Belgium in 2004 for example, 74.3% of women have no disability whilst 25.7% have some disability (see Table 1.1). For the sake of simplicity we shall calculate life expectancy free of any disability but the 25.7% with disability were made up of 9.8% with moderate disability and 15.9% severe disability. We could calculate life expectancy with moderate and with severe disability in a similar way.

Table 1.1 Age-specific prevalence of disability for females in Belgium, 2004

Age group (years)	**Prevalence of disability (%)**	**Age group**	**Prevalence of disability (%)**	**Age group**	**Prevalence of disability (%)**
1-4	4.8	30-34	14.2	60-64	23.4
5-9	3.0	35-39	12.2	65-69	25.7
10-14	7.2	40-44	19.5	70-74	34.5
15-19	9.8	45-49	16.1	75-79	43.1
20-24	8.7	50-54	29.8	80-84	43.1
25-29	9.6	55-59	14.2	85+	51.3

Mortality data

The most important quantities for calculation of life (and therefore health) expectancy are the person years lived in each age group by a future cohort assuming that the same age-specific mortality rates apply. To calculate these we need to know the total time spent in each age group by each member of the cohort. These data are not available, as we do not usually have each individual's life history. Instead we can estimate these using the population in each age group and the number of deaths in the age group. The figures for population and deaths (and indeed often the life tables themselves) are usually obtainable in published form from National Statistics Offices and from Eurostat. Since the morbidity data (prevalence) relate to 2004, we need to use mortality data for the same period.

Calculation of the life table and life expectancy

There are a number of different variants for calculating life tables but the main differences between them are the methods for (i) estimating the probability of survival between birth and age 1, and (ii) closing the life table i.e. the top age considered. We shall use the Eurostat method for (i) (we will explain this later) and a top age group of 85+ as this corresponds to the final age group for the disability prevalence (Table 1.1).

In Tables 1.2 - 1.5 following, the data that is input (e.g. mortality and morbidity data) are shown in italics. These correspond to columns shown in blue in Example 1 on the Excel spreadsheet accompanying the manual. Table 1.2 shows the main life table data required, these being the mid-year population estimates (column[2]) and the number of deaths by single years of age for women in Belgium in 2004 (column[3]).

The results of the steps below to calculate the life table and life expectancy are shown in Table 1.3.

1. The first calculation from the mortality data are the central death rates, m_x (column[4]). Since we have mid-year population estimates (column[2]) and the number of deaths is within one calendar year, the central death rates are calculated as the number of deaths divided by the total population (column[3]/column[2])[2]. If we had had total deaths over a three year period, say 2003-2005 and the mid-interval population (in 2004) then the central death rates would have been calculated as number of deaths divided by the calendar person years (which might be approximated by the mid-interval population multiplied by 3).

2. The probabilities of death in each age interval conditional on having survived to that age, q_x (column[5]) are calculated from the death rates. This is the point in the calculations where the different variations in practice emerge and so we will separate this section into the calculations for the final age group, the first age group and the rest.

   As we have chosen to have the last age group open-ended (85+) we shall not calculate the q_x for the last age. In theory it should be 1 as death during this age group is certain! However this can be very misleading when plotting q_x against age and in fact we shall not require this entry for any further calculations. In the spreadsheet the cell is left blank and shaded grey.

   To calculate the probability of death between birth and age 1 (q_0) we use the formulae used by Eurostat[3]. Some further data are required for this formula, namely the number of births in the year (2004).

   The other entries (apart from the first and last) are calculated as column[4]/(1+0.5*column[4])[4].

3. The third quantity is the number surviving to each age l_x (column[6]). We assume an arbitrary starting number (radix population) of 100,000 at age 0 (birth). We then calculate each successive row in column[6], corresponding to the next age interval, as the previous entry multiplied by its corresponding (1-column[5])[5] . For example, if the number surviving at birth is 100,000 and the probability of dying before age 1 is 0.0036 then we would expect the number surviving to age 1 is 100,000 *(1-0.0036) = 99640. Note that although we have begun the life table at birth, we could have begun the life table at say age 50 years, in which case l_{50} =

[2] See Appendix 1 paragraph 2. for the general formula.
[3] See Appendix 1 paragraph 4. for the general formula.
[4] See Appendix 1 paragraph 3. for the general formula.
[5] See Appendix 1 paragraph 5. for the general formula.

100,000. This would be appropriate if the data for the disability prevalence had been obtained from a survey on the population aged 50 years and over.

4. The number of person years lived at age x, L_x, is calculated in column[7]. At any age (other than the first year of life and the final age interval) this is the average of the entry in column [6] for the same age and the subsequent age interval[6]. Thus the number of person years lived at age 80 is 0.5*(65741.36+62565.73) = 64153.54. If a person survives to the next age then they contribute 1.0 person year. For the first year of life, the number of person years lived in the interval is calculated by a special formula[7]. At age 85+ years the person years lived is the probability of survival to age 85 years (or in our case the number of survivors l_x column[6]) divided by the death rate m_x column[4]) (51850.51/0.186158) = 278530.14[8].

5. Column[8] contains the total number of years lived from the particular age, T_x. This column is calculated by summing all the entries in column[7] from that age to age 85+ years. Hence the total number of years lived from age 80 is given by (64153.54 + 61094.39 + 58343.43 + 56039.53 + 53432.88 + 278530.14) = 571593.91.

6. Finally the total life expectancy at each age e_x (column[9]) is found by dividing the total number of years lived from that age (column[8]) by the probability of surviving to that age (column[6])[9]. Thus in 2004 the female life expectancy at birth in Belgium was 81.4 years.

Calculation of Disability-Free Life Expectancy (DFLE)

Now we need to enter the disability prevalence. We do not have this in single years of age so we match up the prevalence data to the life table data by assuming that each single year of age has the same prevalence as that age group (compare Table 1.1 and Table 1.4 column[10]). We also assume that at birth the prevalence of disability is 0. We could have used more sophisticated methods to get the prevalence by single year from the age grouped data, for instance by fitting a regression model to the five year age groups and using this to estimate the values at single ages.

7. Disability-Free Life Expectancy (DFLE) is found by partitioning the person years lived at that age into those lived with and without disability. To get the person years lived without disability (Table 1.5 column[11]) we multiply the person years lived at that age (column[7]) by the proportion of people <u>without</u> disability at that age (1- column[10]). Since the Belgium survey had a complex design, column[10] contains the weighted rates, that is to say prevalence rates adjusted for the design characteristics.

8. The total number of years lived without disability (column[12]) are found from column[11] in the same way as in paragraph 5 above.

9. Similarly the DFLE at each age (column[13]) is found in the same way as paragraph 6, from column[12] divided by column[6]. Thus women aged 80 years in 2004 could expect to live 8.7 years. A further useful quantity is the proportion of remaining life spent disability-free (Table 1.5 column[14]) which is simply the DFLE divided by the total life expectancy (column[13]/column[9]). Thus women aged 80 years in 2004 could expect to live 52% of their remaining life free of disability.

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[6] See Appendix 1 paragraph 6. for the general formula.
[7] See Appendix 1 paragraph 7. for the general formula.
[8] See Appendix 1 paragraph 8. for the general formula.
[9] See Appendix 1 paragraph 9. for the general formula.

Table 1.2 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single-year life table (method 1)

[1]	*[2]*	*[3]*	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
Age group	***Mid-year population***	***No. deaths***											
x	P_x	D_x											
0	*54795.5*	*202*											
1	*54818*	*21*											
2	*55665.5*	*11*											
3	*55969.5*	*8*											
4	*55805.5*	*12*											
5	*56401.5*	*8*											
…..	…..	…..											
74	*49530.5*	*1059*											
75	*49546.5*	*1161*											
76	*49715*	*1532*											
77	*49928*	*1887*											
78	*48534.5*	*2042*											
79	*45249*	*2121*											
80	*41393.5*	*2049*											
81	*36644*	*1765*											
82	*32714*	*1435*											
83	*28395*	*1038*											
84	*20277.5*	*1201*											
85+	*125152*	*23298*											

Table 1.3 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single-year life table (method 1)

[1]	*[2]*	*[3]*	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
Age group	***Mid-year population***	***No. deaths***	**Central Death rate**	**Conditional probability of death**	**Numbers surviving to age x**	**Person years lived at age x**	**Total number of years lived from age x**	**Total Life Expectancy**					
x	P_x	D_x	m_x	q_x	ℓ_x	L_x	T_x	e_x					
0	*54795.5*	*202*	0.003686	0.003606	100000.00	99711.50	8141517.37	81.4					
1	*54818*	*21*	0.000383	0.000383	99639.37	99620.29	8041805.87	80.7					
2	*55665.5*	*11*	0.000198	0.000198	99601.21	99591.37	7942185.57	79.7					
3	*55969.5*	*8*	0.000143	0.000143	99581.53	99574.41	7842594.20	78.8					
4	*55805.5*	*12*	0.000215	0.000215	99567.30	99556.59	7743019.79	77.8					
5	*56401.5*	*8*	0.000142	0.000142	99545.89	99538.83	7643463.19	76.8					
…..	…..	…..	…..	…..	…..	…..	…..	…..					
74	*49530.5*	*1059*	0.021381	0.021155	80489.15	79637.79	1015838.01	12.6					
75	*49546.5*	*1161*	0.023433	0.023161	78786.43	77874.04	936200.22	11.9					
76	*49715*	*1532*	0.030816	0.030348	76961.65	75793.83	858326.18	11.2					
77	*49928*	*1887*	0.037794	0.037093	74626.01	73241.94	782532.35	10.5					
78	*48534.5*	*2042*	0.042073	0.041206	71857.87	70377.37	709290.40	9.9					
79	*45249*	*2121*	0.046874	0.045801	68896.88	67319.12	638913.03	9.3					
80	*41393.5*	*2049*	0.049501	0.048305	65741.36	64153.54	571593.91	8.7					
81	*36644*	*1765*	0.048166	0.047033	62565.73	61094.39	507440.37	8.1					
82	*32714*	*1435*	0.043865	0.042924	59623.05	58343.43	446345.98	7.5					
83	*28395*	*1038*	0.036556	0.035900	57063.81	56039.53	388002.55	6.8					
84	*20277.5*	*1201*	0.059228	0.057525	55015.25	53432.88	331963.02	6.0					
85+	*125152*	*23298*	0.186158		51850.51	278530.14	278530.14	5.4					

Table 1.4 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single-year life table (method 1)

[1]	*[2]*	*[3]*	[4]	[5]	[6]	[7]	[8]	[9]	*[10]*	[11]	[12]	[13]	[14]
Age	***Mid-year population***	***No. deaths***	**Central Death rate**	**Conditional probability of death**	**Numbers surviving to age x**	**Person years lived at age x**	**Total number of years lived from age x**	**Total Life Expectancy**	***Proportion with disability***				
x	P_x	D_x	m_x	q_x	ℓ_x	L_x	T_x	e_x	π_x				
0	*54795.5*	*202*	0.003686	0.003606	100000.00	99711.50	8141517.37	81.4	*0*				
1	*54818*	*21*	0.000383	0.000383	99639.37	99620.29	8041805.87	80.7	*0.048*				
2	*55665.5*	*11*	0.000198	0.000198	99601.21	99591.37	7942185.57	79.7	*0.048*				
3	*55969.5*	*8*	0.000143	0.000143	99581.53	99574.41	7842594.20	78.8	*0.048*				
4	*55805.5*	*12*	0.000215	0.000215	99567.30	99556.59	7743019.79	77.8	*0.048*				
5	*56401.5*	*8*	0.000142	0.000142	99545.89	99538.83	7643463.19	76.8	*0.030*				
…..	…..	…..	…..	…..	…..	…..	…..	…..					
74	*49530.5*	*1059*	0.021381	0.021155	80489.15	79637.79	1015838.01	12.6	*0.345*				
75	*49546.5*	*1161*	0.023433	0.023161	78786.43	77874.04	936200.22	11.9	*0.431*				
76	*49715*	*1532*	0.030816	0.030348	76961.65	75793.83	858326.18	11.2	*0.431*				
77	*49928*	*1887*	0.037794	0.037093	74626.01	73241.94	782532.35	10.5	*0.431*				
78	*48534.5*	*2042*	0.042073	0.041206	71857.87	70377.37	709290.40	9.9	*0.431*				
79	*45249*	*2121*	0.046874	0.045801	68896.88	67319.12	638913.03	9.3	*0.431*				
80	*41393.5*	*2049*	0.049501	0.048305	65741.36	64153.54	571593.91	8.7	*0.431*				
81	*36644*	*1765*	0.048166	0.047033	62565.73	61094.39	507440.37	8.1	*0.431*				
82	*32714*	*1435*	0.043865	0.042924	59623.05	58343.43	446345.98	7.5	*0.431*				
83	*28395*	*1038*	0.036556	0.035900	57063.81	56039.53	388002.55	6.8	*0.431*				
84	*20277.5*	*1201*	0.059228	0.057525	55015.25	53432.88	331963.02	6.0	*0.431*				
85+	*125152*	*23298*	0.186158		51850.51	278530.14	278530.14	5.4	*0.513*				

Table 1.5 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single-year life table (method 1)

[1]	*[2]*	*[3]*	[4]	[5]	[6]	[7]	[8]	[9]	*[10]*	[11]	[12]	[13]	[14]
Age	***Mid-year population***	***No. deaths***	**Central Death rate**	**Conditional probability of death**	**Numbers surviving to age x**	**Person years lived at age x**	**Total number of years lived from age x**	**Total Life Expectancy**	***Proportion with disability***	**Person years lived without disability at age x**	**Total years lived without disability from age x**	**Disability-free life expectancy**	**Prop. of life spent disability free**
X	P_x	D_x	m_x	q_x	ℓ_x	L_x	T_x	e_x	π_x	$[1-\pi_x]L_x$	$\Sigma[1-\pi_x]L_x$	$DFLE_x$	$\%DFLE/e_x$
0	*54795.5*	*202*	0.003686	0.003606	100000.00	99711.50	8141517.37	81.4	*0*	99711.50	6657315.85	66.6	81.8
1	*54818*	*21*	0.000383	0.000383	99639.37	99620.29	8041805.87	80.7	*0.048*	94838.52	6557604.35	65.8	81.5
2	*55665.5*	*11*	0.000198	0.000198	99601.21	99591.37	7942185.57	79.7	*0.048*	94810.99	6462765.83	64.9	81.4
3	*55969.5*	*8*	0.000143	0.000143	99581.53	99574.41	7842594.20	78.8	*0.048*	94794.84	6367954.85	63.9	81.2
4	*55805.5*	*12*	0.000215	0.000215	99567.30	99556.59	7743019.79	77.8	*0.048*	94777.88	6273160.00	63.0	81.0
5	*56401.5*	*8*	0.000142	0.000142	99545.89	99538.83	7643463.19	76.8	*0.030*	96552.67	6178382.13	62.1	80.8
…..	…..	…..	…..	…..	…..	…..	…..	…..					
74	*49530.5*	*1059*	0.021381	0.021155	80489.15	79637.79	1015838.01	12.6	*0.345*	52162.75	562021.21	7.0	55.3
75	*49546.5*	*1161*	0.023433	0.023161	78786.43	77874.04	936200.22	11.9	*0.431*	44310.33	509858.45	6.5	54.5
76	*49715*	*1532*	0.030816	0.030348	76961.65	75793.83	858326.18	11.2	*0.431*	43126.69	465548.12	6.0	54.2
77	*49928*	*1887*	0.037794	0.037093	74626.01	73241.94	782532.35	10.5	*0.431*	41674.67	422421.43	5.7	54.0
78	*48534.5*	*2042*	0.042073	0.041206	71857.87	70377.37	709290.40	9.9	*0.431*	40044.73	380746.77	5.3	53.7
79	*45249*	*2121*	0.046874	0.045801	68896.88	67319.12	638913.03	9.3	*0.431*	38304.58	340702.04	4.9	53.3
80	*41393.5*	*2049*	0.049501	0.048305	65741.36	64153.54	571593.91	8.7	*0.431*	36503.37	302397.46	4.6	52.9
81	*36644*	*1765*	0.048166	0.047033	62565.73	61094.39	507440.37	8.1	*0.431*	34762.71	265894.10	4.2	52.4
82	*32714*	*1435*	0.043865	0.042924	59623.05	58343.43	446345.98	7.5	*0.431*	33197.41	231131.39	3.9	51.8
83	*28395*	*1038*	0.036556	0.035900	57063.81	56039.53	388002.55	6.8	*0.431*	31886.49	197933.98	3.5	51.0
84	*20277.5*	*1201*	0.059228	0.057525	55015.25	53432.88	331963.02	6.0	*0.431*	30403.31	166047.49	3.0	50.0
85+	*125152*	*23298*	0.186158		51850.51	278530.14	278530.14	5.4	*0.513*	135644.18	135644.18	2.6	48.7

Sullivan health expectancy using an unabridged life table: Example 2

In Example 1 we fitted the prevalence data (five year age groups) to the unabridged life table (single years of age). In this example we will start with the unabridged life table and ‘abridge’ it so that it corresponds to the age groups of the prevalence data. We will use the same data as in Example 1 but will assume steps 1 to 4 have already been made and therefore we will start from where we have calculated the person years lived at age x, L_x.

5. In Table 2.1 column[7a] we will calculate ${}_nL_x$, the number of person years lived **in each age interval**, where n is the length of the interval (usually 5 apart from the first two and the last interval). We do this by summing the individual L_x for each single year of age in the interval. Thus for the age group 1-4 years, n=4 and ${}_4L_1$ = L_1 + L_2 +L_3+ L_4 = 99620.29 + 99591.37 + 99574.41 + 99556.59 = 398342.67.

6. The total number of years lived from age x, T_x (Table 2.2 column[8]), and the total life expectancy, e_x (column[9]), are calculated in the same way as before in steps 5 and 6.

7. In Table 2.3 we show the whole abridged life table with age groups rather than the single years of age and to provide more room for the remaining column we have deleted columns 2-5. The prevalence of disability is added in column[10]. The calculations for DFLE carry through in exactly the same way as in Example 1. Note that e_x and $DFLE_x$ are the values at the beginning of the age interval, for example 12.3 is the number of years spent disability-free at age 65.

Table 2.1 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single-year life table (method 2)

[1]	*[2]*	*[3]*	[4]	[5]	[6]	[7]	[7a]	[9]	[10]	[11]	[12]	[13]	[14]
Age	***Mid-year population***	***No. deaths***	**Central Death rate**	**Conditional probability of death**	**Numbers surviving to age x**	**Person years lived at age x**	**Person years lived in age interval**						
x	P_x	D_x	m_x	q_x	ℓ_x	L_x	${}_nL_x$						
0	*54795.5*	*202*	0.003686	0.003606	100000.00	99711.50	99711.50						
1	*54818*	*21*	0.000383	0.000383	99639.37	99620.29	398342.67						
2	*55665.5*	*11*	0.000198	0.000198	99601.21	99591.37							
3	*55969.5*	*8*	0.000143	0.000143	99581.53	99574.41							
4	*55805.5*	*12*	0.000215	0.000215	99567.30	99556.59							
5	*56401.5*	*8*	0.000142	0.000142	99545.89	99538.83	497564.90						
…..	…..	…..	…..	…..	…..	…..							
74	*49530.5*	*1059*	0.021381	0.021155	80489.15	79637.79							
75	*49546.5*	*1161*	0.023433	0.023161	78786.43	77874.04	364606.31						
76	*49715*	*1532*	0.030816	0.030348	76961.65	75793.83							
77	*49928*	*1887*	0.037794	0.037093	74626.01	73241.94							
78	*48534.5*	*2042*	0.042073	0.041206	71857.87	70377.37							
79	*45249*	*2121*	0.046874	0.045801	68896.88	67319.12							
80	*41393.5*	*2049*	0.049501	0.048305	65741.36	64153.54	293063.77						
81	*36644*	*1765*	0.048166	0.047033	62565.73	61094.39							
82	*32714*	*1435*	0.043865	0.042924	59623.05	58343.43							
83	*28395*	*1038*	0.036556	0.035900	57063.81	56039.53							
84	*20277.5*	*1201*	0.059228	0.057525	55015.25	53432.88							
85+	*125152*	*23298*	0.186158		51850.51	278530.14	278530.14						

Table 2.2 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single-year life table (method 2)

[1]	*[2]*	*[3]*	[4]	[5]	[6]	[7]	[7a]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
Age	***Mid-year population***	***No. deaths***	**Central Death rate**	**Conditional probability of death**	**Numbers surviving to age x**	**Person years lived at age x**	**Person years lived in age interval**	**Total number of years lived from age x**	**Total Life Expectancy**					
x	P_x	D_x	m_x	q_x	ℓ_x	L_x	${}_nL_x$	T_x	e_x					
0	*54795.5*	*202*	0.003686	0.003606	100000.00	99711.50	99711.50	8141517.37	81.4					
1	*54818*	*21*	0.000383	0.000383	99639.37	99620.29	398342.67	8041805.87	80.7					
2	*55665.5*	*11*	0.000198	0.000198	99601.21	99591.37								
3	*55969.5*	*8*	0.000143	0.000143	99581.53	99574.41								
4	*55805.5*	*12*	0.000215	0.000215	99567.30	99556.59								
5	*56401.5*	*8*	0.000142	0.000142	99545.89	99538.83	497564.90	7643463.19	76.8					
…..	…..	…..	…..	…..	…..	…..	…..	…..	…..					
74	*49530.5*	*1059*	0.021381	0.021155	80489.15	79637.79								
75	*49546.5*	*1161*	0.023433	0.023161	78786.43	77874.04	364606.31	936200.22	11.9					
76	*49715*	*1532*	0.030816	0.030348	76961.65	75793.83								
77	*49928*	*1887*	0.037794	0.037093	74626.01	73241.94								
78	*48534.5*	*2042*	0.042073	0.041206	71857.87	70377.37								
79	*45249*	*2121*	0.046874	0.045801	68896.88	67319.12								
80	*41393.5*	*2049*	0.049501	0.048305	65741.36	64153.54	293063.77	571593.91	8.7					
81	*36644*	*1765*	0.048166	0.047033	62565.73	61094.39								
82	*32714*	*1435*	0.043865	0.042924	59623.05	58343.43								
83	*28395*	*1038*	0.036556	0.035900	57063.81	56039.53								
84	*20277.5*	*1201*	0.059228	0.057525	55015.25	53432.88								
85+	*125152*	*23298*	0.186158		51850.51	278530.14	278530.14	278530.14	5.4					

Table 2.3 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using a single-year life table (method 2)

[1]	[1a]	[6]	[7a]	[8]	[9]	[10]	[11]	[12]	[13]	[14]
Age at start of interval	**Age group**	**Numbers surviving to age x**	**Person years lived in age interval**	**Total number of years lived from age x**	**Total Life Expectancy**	***Proportion with disability***	**Person years lived without disability in age interval**	**Total years lived without disability from age x**	**Disability-free life expectancy**	**Proportion of remaining life spent disability-free**
x	**x – x+n**	ℓ_x	$_nL_x$	T_x	e_x	π_x	$[1-\pi_x]_nL_x$	$\Sigma[1-\pi_x]_nL_x$	$DFLE_x$	$\%DFLE_x/e_x$
0	0	100000.00	99711.50	8141517.37	81.4	*0*	99711.50	6657315.85	66.6	81.8
1	1-4	99639.37	398342.67	8041805.87	80.7	*0.048*	379222.22	6557604.35	65.8	81.5
5	5-9	99545.89	497564.90	7643463.19	76.8	*0.03*	482637.95	6178382.13	62.1	80.8
10	10-14	99484.13	497298.40	7145898.30	71.8	*0.072*	461492.91	5695744.17	57.3	79.7
15	15-19	99423.34	496854.73	6648599.90	66.9	*0.098*	448162.96	5234251.26	52.6	78.7
20	20-24	99291.64	496050.19	6151745.17	62.0	*0.087*	452893.83	4786088.30	48.2	77.8
25	25-29	99128.49	495178.43	5655694.98	57.1	*0.096*	447641.31	4333194.47	43.7	76.6
30	30-34	98940.69	494180.05	5160516.54	52.2	*0.089*	450198.02	3885553.16	39.3	75.3
35	35-39	98714.90	492642.40	4666336.50	47.3	*0.142*	422687.18	3435355.14	34.8	73.6
40	40-44	98324.09	490188.26	4173694.09	42.4	*0.122*	430385.29	3012667.96	30.6	72.2
45	45-49	97718.05	486353.79	3683505.83	37.7	*0.195*	391514.80	2582282.67	26.4	70.1
50	50-54	96729.47	479719.40	3197152.04	33.1	*0.161*	402484.57	2190767.87	22.6	68.5
55	55-59	95034.89	470131.64	2717432.64	28.6	*0.298*	330032.41	1788283.29	18.8	65.8
60	60-64	93038.01	458117.83	2247301.01	24.2	*0.234*	350918.26	1458250.88	15.7	64.9
65	65-69	90062.67	440571.98	1789183.18	19.9	*0.257*	327344.98	1107332.62	12.3	61.9
70	70-74	85687.67	412410.98	1348611.20	15.7	*0.345*	270129.19	779987.64	9.1	57.8
75	75-79	78786.43	364606.31	936200.22	11.9	*0.431*	207460.99	509858.45	6.5	54.5
80	80-84	65741.36	293063.77	571593.91	8.7	*0.431*	166753.28	302397.46	4.6	52.9
85	85+	51850.51	278530.14	278530.14	5.4	*0.513*	135644.18	135644.18	2.6	48.7

Sullivan health expectancy using an abridged life table: Example 3

In this example we show for completeness how an abridged life table may be calculated if the death counts or population counts that underlie the life table are only available within age groups.

We take 5-year age groups, apart from the final open-ended group of 85+ that we assume is of length 10 years. Because mortality across the first year of age in not as uniform as across other age intervals we split the first 5-year age group into the single-year age group 0<=age<1 (or as it is written [0,1) and the 4-year age group [1,5).

The same prevalence data will be used as in the previous examples. Table 3.1 shows the data required, these being the mid-year population estimates (column[2]), the number of deaths in 5-year age groups (column[3]) and the prevalence of disability (column[10]).

Calculation of the life table and life expectancy

The results of the steps below to calculate the life table and total life expectancy are shown in Table 3.2.

1. The first calculation from the mortality data are the central death rates, ${}_nm_x$ (column[4]). Since we have mid-year population estimates (column[2]) and the number of deaths is within one calendar year, the central death rates are calculated as the number of deaths divided by the total population (column[3]/column[2])[10].

2. Calculating the death rates within age groups rather than single years of age introduces some bias into the estimate of the survival curve but gives greater precision. We introduce the column a_x, which gives us a picture of the curvature of the true survival curve over the age interval. In Examples 1 and 2 we assumed that $a_x = 0.5$ since for single years of age the assumption that the survival curve can be approximated as a straight line between each year of age is not unreasonable. As we use age groups of five or more years this assumption is less reasonable and values of a_x below 0.5 indicate that the survival curve lies below a straight line over the age interval whilst values of a_x greater than 0.5 indicate that the true curve lies above the straight line. Values of a_x are sometimes available with life tables produced by National Statistical Offices, can be obtained from the World Health Organization or standard calculation methods exist(Chiang 1985). In this example we will simply assume $a_x = 0.5$ and we will show that this is generally a reasonable assumption.

3. From the death rates we calculate the probabilities of death in the age interval conditional on having survived to the beginning of the interval, ${}_nq_x$ (column[5]). As before the last entry is not calculated and the first entry (q_0) is computed with a special formula[11]. The other entries are calculated as n*column[4]/(1 + n*(1-a_x)*column[4])[12] where n is the length of the age group and n = 4 for the second age group and n = 5 for the remainder (except for the final age interval which we assume is of length 10).

4. The calculation of the numbers surviving to the beginning of the age interval, ℓ_x, (column[6]) is the same as Example 1. We assume a radix population of 100,000 at age 0 years and then

[10] See Appendix 1 paragraph 10. for the general formula.
[11] See Appendix 1 paragraph 4. for the general formula.
[12] See Appendix 1 paragraph 10. for the general formula.

calculate each successive row in column[6], corresponding to the next age interval, as the previous entry multiplied by its corresponding (1-column[5])[13].

5. The number of person years lived **in each age interval**, ${}_nL_x$, is calculated in column[7]. This is found by multiplying the entry in column[6] for the same age group by $n*a_x$ and adding this to $n*(1-a_x)$ multiplied by the entry in column[6] for the next age group, where n is the size of the age group[14]. Thus for the person years lived in the 30-34 year age group, ${}_5L_{30}$, we have n = 5, a_x = 0.5, ℓ_{30} = 0.9900 and ℓ_{35} = 98940.5. Thus ${}_5L_{30}$ = 5*0.5*98940.5 + 5*0.5*98714.1 = 494136.5. For the first year of life, the number of person years lived in the interval [0,1) is calculated by a special formula[15]. As in Example 1, the person years lived at age 85+ is the numbers surviving to age 85 years divided by the death rate (51976.2/0.186158) = 279205.1[16].

6. From this point onwards the calculations carry through as in Example 1. We can see the very small effect that using a=0.5 has on the life expectancies e_x.

Calculation of Disability-Free Life Expectancy (DFLE)

Table 3.3 shows the final steps below to calculate DFLE which carry through in the same way as previous examples. Comparison with the previous examples shows that the effect on DFLE of using a=0.5 is very small – at the most in the second decimal place. In Column[14] we also show the proportion of remaining life spent disability-free as calculated in Example 1 (section 9).

[13] See Appendix 1 paragraph 5. for the general formula.
[14] See Appendix 1 paragraph 10. for the general formula.
[15] See Appendix 1 paragraph 7. for the general formula.
[16] See Appendix 1 paragraph 8. for the general formula.

Table 3.1 Data for the calculation of Disability-Free Life Expectancy (DFLE) using an abridged life table

[1]	[1a]	*[2]*	*[3]*	[4]		[5]	[6]	[7]	[8]	[9]	*[10]*	[11]	[12]	[13]
Age at start of interval	**Age group**	***Mid-year population***	***No. deaths***								***Proportion of age group with disability***			
x	**x – x+n**	P_x	D_x								π_x			
0	0	*54795.5*	*202*								*0.000*			
1	1-4	*222258.5*	*52*								*0.048*			
5	5-9	*290541.0*	*36*								*0.030*			
10	10-14	*310760.0*	*38*								*0.072*			
15	15-19	*297484.5*	*79*								*0.098*			
20	20-24	*315530.0*	*104*								*0.087*			
25	25-29	*319059.5*	*121*								*0.096*			
30	30-34	*357971.0*	*164*								*0.089*			
35	35-39	*393382.5*	*313*								*0.142*			
40	40-44	*403856.5*	*499*								*0.122*			
45	45-49	*377145.0*	*764*								*0.195*			
50	50-54	*357821.0*	*1264*								*0.161*			
55	55-59	*311235.5*	*1323*								*0.298*			
60	60-64	*248213.5*	*1610*								*0.234*			
65	65-69	*261119.0*	*2600*								*0.257*			
70	70-74	*262491.0*	*4390*								*0.345*			
75	75-79	*242973.0*	*8743*								*0.431*			
80	80-84	*159424.0*	*7488*								*0.431*			
85	85+	*125152.0*	*23298*								*0.513*			

Table 3.2 Calculation of the life table quantities for DFLE by the Sullivan method using an abridged life table

[1] Age at start of interval x	[1a] Age group x – x+n	*[2] Mid-year population* P_x	*[3] No. deaths* D_x	[4] Central Death rate $_nm_x$	a_x	[5] Conditional probability of death in age interval $_nq_x$	[6] Numbers surviving to age x ℓ_x	[7] Person years lived in age interval $_nL_x$	[8] Total number of years lived from age x T_x	[9] Total Life Expectancy e_x	*[10] Proportion of age group with disability* π_x	[11]	[12]	[13]
0	0	*54795.5*	*202*	0.003686	*0.5*	0.00360626	100000.0	99711.5	8137192.9	81.4	*0.000*			
1	1-4	*222258.5*	*52*	0.000234	*0.5*	0.00093541	99639.4	398371.1	8037481.4	80.7	*0.048*			
5	5-9	*290541.0*	*36*	0.000124	*0.5*	0.00061934	99546.2	497576.7	7639110.3	76.7	*0.030*			
10	10-14	*310760.0*	*38*	0.000122	*0.5*	0.00061122	99484.5	497270.6	7141533.6	71.8	*0.072*			
15	15-19	*297484.5*	*79*	0.000266	*0.5*	0.00132692	99423.7	496788.7	6644263.0	66.8	*0.098*			
20	20-24	*315530.0*	*104*	0.000330	*0.5*	0.00164666	99291.8	496050.2	6147474.3	61.9	*0.087*			
25	25-29	*319059.5*	*121*	0.000379	*0.5*	0.00189440	99128.3	495171.9	5651424.1	57.0	*0.096*			
30	30-34	*357971.0*	*164*	0.000458	*0.5*	0.00228807	98940.5	494136.5	5156252.2	52.1	*0.089*			
35	35-39	*393382.5*	*313*	0.000796	*0.5*	0.00397042	98714.1	492590.7	4662115.6	47.2	*0.142*			
40	40-44	*403856.5*	*499*	0.001236	*0.5*	0.00615891	98322.2	490097.0	4169524.9	42.4	*0.122*			
45	45-49	*377145.0*	*764*	0.002026	*0.5*	0.01007769	97716.6	486121.2	3679427.9	37.7	*0.195*			
50	50-54	*357821.0*	*1264*	0.003532	*0.5*	0.01750785	96731.9	479425.4	3193306.7	33.0	*0.161*			
55	55-59	*311235.5*	*1323*	0.004251	*0.5*	0.02103051	95038.3	470194.7	2713881.4	28.6	*0.298*			
60	60-64	*248213.5*	*1610*	0.006486	*0.5*	0.03191424	93039.6	457774.7	2243686.7	24.1	*0.234*			
65	65-69	*261119.0*	*2600*	0.009957	*0.5*	0.04857652	90070.3	439413.3	1785911.9	19.8	*0.257*			
70	70-74	*262491.0*	*4390*	0.016724	*0.5*	0.08026592	85695.0	411279.0	1346498.7	15.7	*0.345*			
75	75-79	*242973.0*	*8743*	0.035983	*0.5*	0.16506785	78816.6	361557.8	935219.6	11.9	*0.431*			
80	80-84	*159424.0*	*7488*	0.046969	*0.5*	0.21016706	65806.5	294456.7	573661.8	8.7	*0.431*			
85	85+	*125152.0*	*23298*	0.186158	*0.5*		51976.2	279205.1	279205.1	5.4	*0.513*			

Table 3.3 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using an abridged life table

[1]	[1a]		[6]	[7]	[8]	[9]	*[10]*	[11]	[12]	[13]	[14]
Age at start of interval	**Age group**		**Numbers surviving to age x**	**Person years lived in age interval**	**Total number of years lived from age x**	**Total Life Expectancy**	***Proportion of age group with disability***	**Person years lived without disability in interval**	**Total years lived without disability from age x**	**Disability-free life expectancy**	**Proportion of remaining life spent disability-free**
x	**x – x+n**	a_x	ℓ_x	$_nL_x$	T_x	e_x	π_x	$(1-\pi_x)*L_x$	$\Sigma[(1-\pi_x)*L_x]$	$DFLE_x$	$\%DFLE_x/ e_x$
0	0	*0.5*	100000.0	99711.5	8137192.9	81.4	*0.000*	99711.5	6654230.9	**66.5**	**81.8**
1	1-4	*0.5*	99639.4	398371.1	8037481.4	80.7	*0.048*	379249.3	6554519.4	**65.8**	**81.5**
5	5-9	*0.5*	99546.2	497576.7	7639110.3	76.7	*0.030*	482649.4	6175270.1	**62.0**	**80.8**
10	10-14	*0.5*	99484.5	497270.6	7141533.6	71.8	*0.072*	461467.1	5692620.7	**57.2**	**79.7**
15	15-19	*0.5*	99423.7	496788.7	6644263.0	66.8	*0.098*	448103.4	5231153.6	**52.6**	**78.7**
20	20-24	*0.5*	99291.8	496050.2	6147474.3	61.9	*0.087*	452893.8	4783050.2	**48.2**	**77.8**
25	25-29	*0.5*	99128.3	495171.9	5651424.1	57.0	*0.096*	447635.4	4330156.3	**43.7**	**76.6**
30	30-34	*0.5*	98940.5	494136.5	5156252.2	52.1	*0.089*	450158.4	3882520.9	**39.2**	**75.3**
35	35-39	*0.5*	98714.1	492590.7	4662115.6	47.2	*0.142*	422642.8	3432362.5	**34.8**	**73.6**
40	40-44	*0.5*	98322.2	490097.0	4169524.9	42.4	*0.122*	430305.2	3009719.7	**30.6**	**72.2**
45	45-49	*0.5*	97716.6	486121.2	3679427.9	37.7	*0.195*	391327.6	2579414.6	**26.4**	**70.1**
50	50-54	*0.5*	96731.9	479425.4	3193306.7	33.0	*0.161*	402237.9	2188087.0	**22.6**	**68.5**
55	55-59	*0.5*	95038.3	470194.7	2713881.4	28.6	*0.298*	330076.7	1785849.1	**18.8**	**65.8**
60	60-64	*0.5*	93039.6	457774.7	2243686.7	24.1	*0.234*	350655.4	1455772.4	**15.6**	**64.9**
65	65-69	*0.5*	90070.3	439413.3	1785911.9	19.8	*0.257*	326484.0	1105117.0	**12.3**	**61.9**
70	70-74	*0.5*	85695.0	411279.0	1346498.7	15.7	*0.345*	269387.8	778632.9	**9.1**	**57.8**
75	75-79	*0.5*	78816.6	361557.8	935219.6	11.9	*0.431*	205726.4	509245.2	**6.5**	**54.5**
80	80-84	*0.5*	65806.5	294456.7	573661.8	8.7	*0.431*	167545.9	303518.8	**4.6**	**52.9**
85	85+	*0.5*	51976.2	279205.1	279205.1	5.4	*0.513*	135972.9	135972.9	**2.6**	**48.7**

The standard error of the Sullivan health expectancy: Example 4

The prevalence of disability by single- or five-year age groups shows considerable fluctuation due to sampling variation. Mortality rates are also subject to random variation. Since the Sullivan health expectancy combines such mortality and morbidity rates, it too is subject to random variation. To assess the size of this random variation, we shall calculate the standard error of DFLE for females in Belgium in 2004 from the abridged life table in Example 3.

If the sample size of the survey producing the prevalence ratios is not very large compared to the population on which the mortality data are based, then the variation resulting from the mortality rates is negligible and this part of the variance can be ignored (Newman 1988). This will be shown in the following example. First the standard errors will be calculated by taking the variance of the prevalence rates only into account (Table 4.1). In Table 4.2 the standard errors will be recalculated by adding the part of the variance resulting from the mortality rates.

Approximate standard errors ignoring the variance of the mortality rates

The calculations continue from Example 3 (shown in Table 3.3) and the column numbers for the first 9 columns in Table 4.1 correspond to those in Table 3.3. For clarity we omit columns [2]-[5] and [8] from Table 4.1.

1. The first extra column (column[15]) in Table 4.1 is the number of persons in the age interval who took part in the survey. If the survey does not have a complex design and if the non response rates are not high, then column[15] contains the denominators used to calculate the prevalence in column[10].

2. The variance of the prevalence rates is in column[16] and is calculated as Column[16] = {column[10]*(1-column[10])}/column[15][17]. If the survey providing the prevalence rates has a complex sampling design then the appropriate Statistical Institute involved in the survey will provide more accurate estimates of the variances of the prevalence rates. In case these more accurate estimates are not available, a simple approximation to the calculation of the standard error of DFLE is to use the general formula above but with the <u>weighted prevalence</u> in column[10] and in column[15] the <u>unweighted</u> number of persons in the age interval i.e. the <u>actual</u> number who took part in the survey in the given age interval.

3. In column[18] we will calculate $\sum_{x=0}^{W} L_x^2 S^2(\pi_x)$ so we do an intermediate calculation to first obtain the individual $L_x^2 S^2$ in column[17] as Column[17] = column[7]*column[7]*column[16].

4. Column[18] is then found by summing the entries in column[17] from that age interval to the final age interval (in this case 85+).

5. The variance of the health expectancy is given in column[19] where column[19] = column[18]/(column[6]*column [6]).

6. The standard error of the health expectancy in column[20] is the square root of column[19].

[17] See Appendix 1 paragraph 11. for the general formula.

If required, approximate 95% confidence intervals for the DFLE are given by column[13] - 1.96*column[20] and column[13]+1.96*column[20].

Standard errors taking into account the variance of the mortality rates

Again the column numbers of the first 7 columns of table 4.2 correspond to those in table 3.3. Column[19] contains the variance of DFLE as calculated in table 4.1 and will be renamed as $S^2_{(1)}(DFLE_x)$ – this is the second term in the formula shown in Appendix 1 paragraph 11 [18]. The part of the variance resulting from the mortality rates will be named $S^2_{(2)}(DFLE_x)$ and this is the first term in the formula. We will denote the total variance by $S^2(DFLE_x)$.

1. For $S^2_{(2)}(DFLE_x)$ we need to calculate $\ell_x^2[(1- a_x)n(1-\pi_x) + DFLE_{x+n}]^2 S^2(p_x)$. We shall do this in two parts: first calculating $S^2(p_x)$ in column[21] and then $[(1- a_x)n(1-\pi_x) + DFLE_{x+n}]$ in column[22].

2. The variance of the probability of death, $S^2(p_x)$ which we shall calculate in column[21], is identical to the variance of the probability of survival, $S^2(q_x)$. Thus Column[21] = {column[5]*column[5]*(1-column[5])}/column[3]. For the last age group 85+ this part is in fact 0 (since the conditional probability of death q_x is 1) but we have again omitted it.

3. $[(1- a_x)n(1-\pi_x) + DFLE_{x+n}]$ is calculated in column[22] by adding $\{(1-a_x)*n*(1-$column[10])} for an age group to the entry in column[13] for the next age group. Remember n is the length of the age interval and is therefore 1 for the first age group, 2 for the second and five for the remainder. For example for the 30-34 year age group we have $a_{30} = 0.5$, $n = 5$, $\pi_x = 0.089$ and $DFLE_{35} = 34.8$, thus column[22] = {(1-0.5)*5*(1-0.089)}+34.8 = 37.0482.

4. Now we calculate $\ell_x^2[(1- a_x)n(1-\pi_x) + DFLE_{x+n}]^2 S^2(p_x)$ in column[23] as column[23]=column[22]*column[22]*column[6]*column[6]*column[21].

5. We now need to sum column[23] from age x to age 85+ years and the result is shown in column[24].

6. The part of the variance resulting from the mortality rates, $S^2_{(2)}(DFLE_x)$, is shown in column[25] and equals column[24]/(column[6]*column[6]).

7. Column[26] contains the total variance, $S^2(DFLE_x)$, which is the sum of column[19] and column[25].

The example shows that the part of the variance resulting from the mortality rates is only a very small part of the total variance and can be ignored in this case.

[18] See Appendix 1 paragraph 11. for the general formula.

Table 4.1 Calculation of the standard error for DFLE by the Sullivan method using an abridged life table, ignoring the variance from the mortality rates

[1]	[1a]	[6]	[7]	[9]	*[10]*	[11]	[12]	[13]	*[15]*	[16]	[17]	[18]	[19]	[20]
Age at start of interval	**Age group**	**Numbers surviving to age x**	**Person years lived in age interval**	**Total Life Expectancy**	***Proportion of age group with disability***	**Person years lived without disability in interval**	**Total years lived without disability from age x**	**Disability-free life expectancy**	***Number in survey in age interval***				**Variance of $DFLE_x$**	**Standard error of $DFLE_x$**
x	**x – x+n**	ℓ_x	${}_nL_x$	e_x	π_x	$(1-\pi_x)*L_x$	$\Sigma[(1-\pi_x)*L_x]$	$DFLE_x$	N_x	$S^2(\pi_x)$	$L^2S^2(\pi_x)$	$\Sigma L^2S^2(\pi_x)$	$S^2(DFLE_x)$	$S(DFLE_x)$
0	0	100000.0	99711.5	81.4	*0.000*	99711.5	6654230.9	**66.5**	*52*	0.000000	0	1261478651	0.12615	**0.355**
1	1-4	99639.4	398371.1	80.7	*0.048*	379249.3	6554519.4	**65.8**	*230*	0.000197	31530146	1261478651	0.12706	**0.356**
5	5-9	99546.2	497576.7	76.7	*0.030*	482649.4	6175270.1	**62.0**	*257*	0.000112	28033671	1229948505	0.12412	**0.352**
10	10-14	99484.5	497270.6	71.8	*0.072*	461467.1	5692620.7	**57.2**	*275*	0.000243	60080466	1201914834	0.12144	**0.348**
15	15-19	99423.7	496788.7	66.8	*0.098*	448103.4	5231153.6	**52.6**	*281*	0.000314	77637184	1141834368	0.11551	**0.340**
20	20-24	99291.8	496050.2	61.9	*0.087*	452893.8	4783050.2	**48.2**	*342*	0.000231	57149854	1064197184	0.10794	**0.329**
25	25-29	99128.3	495171.9	57.0	*0.096*	447635.4	4330156.3	**43.7**	*402*	0.000215	52932898	1007047330	0.10248	**0.320**
30	30-34	98940.5	494136.5	52.1	*0.089*	450158.4	3882520.9	**39.2**	*426*	0.000190	46472141	954114432	0.09747	**0.312**
35	35-39	98714.1	492590.7	47.2	*0.142*	422642.8	3432362.5	**34.8**	*437*	0.000279	67649820	907642291	0.09314	**0.305**
40	40-44	98322.2	490097.0	42.4	*0.122*	430305.2	3009719.7	**30.6**	*446*	0.000241	57687743	839992471	0.08689	**0.295**
45	45-49	97716.6	486121.2	37.7	*0.195*	391327.6	2579414.6	**26.4**	*438*	0.000358	84692606	782304728	0.08193	**0.286**
50	50-54	96731.9	479425.4	33.0	*0.161*	402237.9	2188087.0	**22.6**	*424*	0.000318	73225784	697612123	0.07455	**0.273**
55	55-59	95038.3	470194.7	28.6	*0.298*	330076.7	1785849.1	**18.8**	*407*	0.000514	113635608	624386339	0.06913	**0.263**
60	60-64	93039.6	457774.7	24.1	*0.234*	350655.4	1455772.4	**15.6**	*310*	0.000578	121167615	510750731	0.05900	**0.243**
65	65-69	90070.3	439413.3	19.8	*0.257*	326484.0	1105117.0	**12.3**	*398*	0.000480	92637146	389583116	0.04802	**0.219**
70	70-74	85695.0	411279.0	15.7	*0.345*	269387.8	778632.9	**9.1**	*413*	0.000547	92551503	296945970	0.04044	**0.201**
75	75-79	78816.6	361557.8	11.9	*0.431*	205726.4	509245.2	**6.5**	*310*	0.000791	103414969	204394467	0.03290	**0.181**
80	80-84	65806.5	294456.7	8.7	*0.431*	167545.9	303518.8	**4.6**	*300*	0.000817	70877957	100979498	0.02332	**0.153**
85	85+	51976.2	279205.1	5.4	*0.513*	135972.9	135972.9	**2.6**	*647*	0.000386	30101542	30101542	0.01114	**0.106**

Table 4.2 Calculation of the standard error for DFLE by the Sullivan method using an abridged life table, taking into account the variance from the mortality rates

[1]	[1a]		*[3]*	[5]	[6]	*[10]*	[13]	[19]	[21]	[22]	[23]	[24]	[25]	[26]
Age at start of interval	**Age group**		***No. deaths***	**Conditional probability of death in age interval**	**Numbers surviving to age x**	***Proportion of age group with disability***	**Disability-free life expectancy**	**Variance of DFLE (due to prevalence)**					**Variance of DFLE (due to mortality)**	**Total Variance of DFLE**
x	**x – x+n**	a_x	D_x	$_nq_x$	ℓ_x	π_x	**DFLE**$_x$	$S_{(1)}^2(DFLE_x)$	$S^2(q_x)$				$S^2_{(2)}(DFLE_x)$	$S^2(DFLE_x)$
0	0	*0.5*	*202*	0.003606	100000.0	*0.000*	**66.5**	0.12615	0.00000006	66.2824	2818317.73	13425392.5	0.00134	0.12749
1	1-4	*0.5*	*52*	0.00093541	99639.4	*0.048*	**65.8**	0.12706	0.00000002	63.9382	682302.568	10607074.8	0.00107	0.12813
5	5-9	*0.5*	*36*	0.00061934	99546.2	*0.030*	**62.0**	0.12412	0.00000001	59.6462	375408.278	9924772.19	0.00100	0.12512
10	10-14	*0.5*	*38*	0.00061122	99484.5	*0.072*	**57.2**	0.12144	0.00000001	54.9347	293459.144	9549363.91	0.00096	0.12241
15	15-19	*0.5*	*79*	0.00132692	99423.7	*0.098*	**52.6**	0.11551	0.00000002	50.4267	559481.406	9255904.76	0.00094	0.11645
20	20-24	*0.5*	*104*	0.00164666	99291.8	*0.087*	**48.2**	0.10794	0.00000003	45.9648	542174.627	8696423.36	0.00088	0.10883
25	25-29	*0.5*	*121*	0.00189440	99128.3	*0.096*	**43.7**	0.10248	0.00000003	41.5010	501010.749	8154248.73	0.00083	0.10331
30	30-34	*0.5*	*164*	0.00228807	98940.5	*0.089*	**39.2**	0.09747	0.00000003	37.0482	427939.469	7653237.98	0.00078	0.09825
35	35-39	*0.5*	*313*	0.00397042	98714.1	*0.142*	**34.8**	0.09314	0.00000005	32.7558	524487.537	7225298.51	0.00074	0.09389
40	40-44	*0.5*	*499*	0.00615891	98322.2	*0.122*	**30.6**	0.08689	0.00000008	28.5919	597053.222	6700810.98	0.00069	0.08758
45	45-49	*0.5*	*764*	0.01007769	97716.6	*0.195*	**26.4**	0.08193	0.00000013	24.6326	762409.587	6103757.75	0.00064	0.08257
50	50-54	*0.5*	*1264*	0.01750785	96731.9	*0.161*	**22.6**	0.07455	0.00000024	20.8883	972734.46	5341348.17	0.00057	0.07513
55	55-59	*0.5*	*1323*	0.02103051	95038.3	*0.298*	**18.8**	0.06913	0.00000033	17.4018	895148.044	4368613.71	0.00048	0.06961
60	60-64	*0.5*	*1610*	0.03191424	93039.6	*0.234*	**15.6**	0.05900	0.00000061	14.1845	1066645.72	3473465.66	0.00040	0.05940
65	65-69	*0.5*	*2600*	0.04857652	90070.3	*0.257*	**12.3**	0.04802	0.00000086	10.9436	838951.023	2406819.94	0.00030	0.04832
70	70-74	*0.5*	*4390*	0.08026592	85695.0	*0.345*	**9.1**	0.04044	0.00000135	8.0986	650122.626	1567868.92	0.00021	0.04065
75	75-79	*0.5*	*8743*	0.16506785	78816.6	*0.431*	**6.5**	0.03290	0.00000260	6.0348	588674.843	917746.291	0.00015	0.03305
80	80-84	*0.5*	*7488*	0.21016706	65806.5	*0.431*	**4.6**	0.02332	0.00000466	4.0386	329071.447	329071.447	0.00008	0.02339
85	85+	*0.5*	*23298*	[redacted]	51976.2	*0.513*	**2.6**	0.01114	[redacted]	2.4350	[redacted]			0.01114

Testing the equality of two Sullivan health expectancies: Example 5

To test whether two Sullivan health expectancies are equal, we need the two health expectancies and their standard errors (or variances). We shall compare the Disability-Free Life Expectancy (DFLE) between females and males in Belgium in 2004. The test of the hypothesis of equality of DFLE for females and males in 2004 is shown in Table 5.

1. Column[2] and column[3] of Table 5 are the DFLE for females in Belgium in 2004 ($DFLE_{(F)}$) and the standard error. These were already calculated in column[13] and column[20] of Table 4.1. The equivalent values for males are shown in column[4] ($DFLE_{(M)}$) and column[5] of Table 5.

2. Column[6] shows the difference in DFLE between females and males and is found by subtracting column[4] from column[2].

3. The <u>approximate</u> standard error of this difference is shown in column[7] and is found by adding together column[3] and column[5][19].

4. The z-statistic in column[8] is formed from column[6]/column[7]. If a one-sided test is required e.g. testing that, at a particular age, $DFLE_{(F)} > DFLE_{(M)}$ then the p-value (column[9]) is the probability that a standard normal variate exceeds the value in column[8]. If a two-sided test is required, simply testing that a difference exists in DFLE between females and males then the p-value (column[9]) is twice the probability that a standard normal variate exceeds the value in column[8]. These probabilities are found from tables of the standard normal distribution (see Appendix 2).

The results show that in Belgium, Disability-Free Life Expectancy (DFLE) is significantly different between females and males.

There are two other issues worth discussing in this example because they are particularly relevant for comparisons between groups, times or geographical areas. First the number of people at the oldest ages may well be small and this is particularly true with national health surveys. Thus a large final age group is required to give sufficient precision to estimates of this age group. However, the prevalence of disability increases exponentially with age and therefore assuming a constant prevalence for this large open-ended age group will produce biased estimates. Fortunately, if we are comparing two points in time or two geographical areas, the same bias will apply to both. Thus estimates of the **difference** in DFLE will be unbiased.

Secondly, if estimates of DFLE for different regions in a country are to be compared, then the numbers in many age groups may be small within regions. To account for this we may wish to create an abridged life table with wider than 5 year age groups, perhaps say 15 or 20 year age groups i.e. 0-14, 15-24, 25-44, 45-64, 65+. As we did in Example 2 we can calculate the L_x for the abridged table by summing values for single years of age (or smaller age groups). Hence L_{15-24} can be calculated as $L_{15-24} = L_{15} + L_{16} + ... + L_{24}$ or $L_{15-24} = L_{15-19} + L_{20-24}$.

Again, although assuming a constant prevalence over large age groups will introduce a bias, the bias will be present for all regions therefore making the differences in DFLE between regions unbiased.

[19] See Appendix 1 paragraph 12. for the general formula.

Table 5 Comparisons of Disability-Free Life Expectancy between males ($DFLE_{(M)}$) and females ($DFLE_{(F)}$)

[1]	[1a]	*[2]*	*[3]*	*[4]*	*[5]*	[6]	[7]	[8]	[9]
Age at start of interval	**Age group**	**Disability-free life expectancy females**	**Standard error of DFLE females**	**Disability-free life expectancy males**	**Standard error of DFLE males**	**Difference in DFLE between males and females**	**Approximate standard error of difference in DFLE between males and females**	**z statistic**	**p value Pr(Z≥|z|)**
x	**x – x+n**	$DFLE_{(F)}$	$S(DFLE_{(F)})$	$DFLE_{(M)}$	$S(DFLE_{(M)})$	$DFLE_{(F)} - DFLE_{(M)}$	$S(DFLE_{(F)}) + S(DFLE_{(M)})$	**z**	**p**
0	0	*66.5*	*0.36*	*63.5*	*0.33*	3.07	0.69	4.47	<0.001
1	1-4	*65.8*	*0.36*	*62.7*	*0.33*	3.04	0.69	4.42	<0.001
5	5-9	*62.0*	*0.35*	*58.9*	*0.33*	3.12	0.68	4.57	<0.001
10	10-14	*57.2*	*0.35*	*54.2*	*0.32*	3.00	0.67	4.47	<0.001
15	15-19	*52.6*	*0.34*	*49.6*	*0.32*	3.02	0.65	4.61	<0.001
20	20-24	*48.2*	*0.33*	*45.0*	*0.31*	3.21	0.64	5.03	<0.001
25	25-29	*43.7*	*0.32*	*40.5*	*0.31*	3.22	0.63	5.15	<0.001
30	30-34	*39.2*	*0.31*	*36.0*	*0.30*	3.21	0.61	5.25	<0.001
35	35-39	*34.8*	*0.31*	*31.7*	*0.29*	3.08	0.60	5.16	<0.001
40	40-44	*30.6*	*0.29*	*27.5*	*0.28*	3.12	0.58	5.40	<0.001
45	45-49	*26.4*	*0.29*	*23.8*	*0.27*	2.56	0.56	4.60	<0.001
50	50-54	*22.6*	*0.27*	*20.1*	*0.26*	2.53	0.53	4.78	<0.001
55	55-59	*18.8*	*0.26*	*16.9*	*0.24*	1.93	0.50	3.84	<0.001
60	60-64	*15.6*	*0.24*	*13.5*	*0.23*	2.13	0.47	4.52	<0.001
65	65-69	*12.3*	*0.22*	*10.6*	*0.21*	1.66	0.43	3.90	<0.001
70	70-74	*9.1*	*0.20*	*7.8*	*0.20*	1.32	0.40	3.32	<0.001
75	75-79	*6.5*	*0.18*	*5.4*	*0.19*	1.02	0.37	2.75	<0.01
80	80-84	*4.6*	*0.15*	*3.9*	*0.18*	0.71	0.33	2.17	<0.05
85	85+	*2.6*	*0.11*	*2.6*	*0.16*	0.01	0.26	0.02	>0.20

Sullivan health expectancy using separate data sources for institutionalised: Example 6

The estimates of life expectancy are made from total population data thus including those in institutions. However survey data for the prevalence of the health states commonly do not include the institutionalised population. Since rates of institutionalisation differ both between countries and within countries over time, using surveys that omit those in institutions for the calculation of prevalence of health states can seriously bias comparisons. We now demonstrate how information on the institutionalised population (who were omitted from the Belgian health survey) can be incorporated to produce the prevalence for the total Belgian population.

The most common and simplest method used to incorporate the institutionalised population into health expectancy estimates assumes that all those in institutions are in a particular health state (in our example we assume they all have disability). The only extra data that are required are therefore the proportion of the total population who are in institutions by age group. These data are usually obtained from Census figures.

Calculation of Disability-Free Life Expectancy (DFLE)

1. We show in Table 6.1 columns[6], [7], [9] and [10] from Table 3.3 (Example 3) as these will be required for the calculations.

2. The extra data needed is the proportion in institutions in each age group, I_x which was obtained from Census figures.

3. The new proportion disabled in the total population (non-institutionalised and institutionalized), π'_x, is calculated in column[10a] as Column[10a] = (1 - I_x)* column[10] + I_x [20]. The remainder of the calculation of DFLE continues as in Example 3. To see the effect of including those in institutions we add in the DFLE from Example 3 (column[13]). Note that the DFLE decreases for females when the numbers in institutions is taken into account.

Approximate standard errors ignoring the variance of the mortality rates

4. To calculate the new standard error we will again need the number of people interviewed in the survey in each age group, N_x and this is in column[15] (remember we used it in Example 4 Table 4.1).

5. Since we have new prevalence rates including those in institutions this will change the variance of the prevalence rates which we calculated in column[16] in Example 4 step 2. The previous calculation should be replaced by Column[16] = (1 - I_x)*(1 - I_x)*column[10a]*(1-column[10a])/column[15][21].

The calculation of the standard error of $DFLE_x$ continues in the same way as in Example 4 from step 3 onward.

[20] See Appendix 1 paragraph 13. for the general formula.
[21] See Appendix 1 paragraph 14. for the general formula.

Table 6.1 Calculation of Disability-Free Life Expectancy (DFLE) by the Sullivan method using an abridged life table with institutionalized population

[1] Age at start of interval x	[1a] Age group $x - x+n$	[6] Numbers surviving to age x ℓ_x	[7] Person years lived in age interval ${}_nL_x$	[9] Total Life Expectancy e_x	*[10] Proportion of age group in survey with disability* π_x	*Proportion in institutions* I_x	[10a] Proportion of age group (total) with disability π'_x	[11a] Person years lived without disability in interval $(1-\pi'_x)*L_x$	[12a] Total years lived without disability from age x $\Sigma[(1-\pi'_x)*L_x]$	[13a] Disability-free life expectancy (including inst) $DFLE'_x$	[13] Disability-free life expectancy (excluding inst) $DFLE_x$
0	0	100000.0	99711.5	81.4	*0.000*	*0.000*	0.000	99711.5	6570426.4	**65.7**	**66.5**
1	1-4	99639.4	398371.1	80.7	*0.048*	*0.000*	0.048	379249.3	6470714.9	**64.9**	**65.8**
5	5-9	99546.2	497576.7	76.7	*0.030*	*0.000*	0.030	482649.4	6091465.7	**61.2**	**62.0**
10	10-14	99484.5	497270.6	71.8	*0.072*	*0.000*	0.072	461467.1	5608816.2	**56.4**	**57.2**
15	15-19	99423.7	496788.7	66.8	*0.098*	*0.000*	0.098	448103.4	5147349.2	**51.8**	**52.6**
20	20-24	99291.8	496050.2	61.9	*0.087*	*0.001*	0.087	452893.8	4699245.7	**47.3**	**48.2**
25	25-29	99128.3	495171.9	57.0	*0.096*	*0.001*	0.097	447187.8	4246351.9	**42.8**	**43.7**
30	30-34	98940.5	494136.5	52.1	*0.089*	*0.001*	0.090	449708.2	3799164.1	**38.4**	**39.2**
35	35-39	98714.1	492590.7	47.2	*0.142*	*0.001*	0.143	422220.2	3349455.9	**33.9**	**34.8**
40	40-44	98322.2	490097.0	42.4	*0.122*	*0.002*	0.124	429444.5	2927235.7	**29.8**	**30.6**
45	45-49	97716.6	486121.2	37.7	*0.195*	*0.002*	0.197	390544.9	2497791.2	**25.6**	**26.4**
50	50-54	96731.9	479425.4	33.0	*0.161*	*0.003*	0.164	401031.2	2107246.3	**21.8**	**22.6**
55	55-59	95038.3	470194.7	28.6	*0.298*	*0.003*	0.300	329086.5	1706215.1	**18.0**	**18.8**
60	60-64	93039.6	457774.7	24.1	*0.234*	*0.006*	0.239	348551.5	1377128.6	**14.8**	**15.6**
65	65-69	90070.3	439413.3	19.8	*0.257*	*0.009*	0.264	323545.7	1028577.1	**11.4**	**12.3**
70	70-74	85695.0	411279.0	15.7	*0.345*	*0.019*	0.357	264269.4	705031.4	**8.2**	**9.1**
75	75-79	78816.6	361557.8	11.9	*0.431*	*0.047*	0.458	196057.3	440762.0	**5.6**	**6.5**
80	80-84	65806.5	294456.7	8.7	*0.431*	*0.110*	0.494	149115.8	244704.8	**3.7**	**4.6**
85	85+	51976.2	279205.1	5.4	*0.513*	*0.297*	0.658	95588.9	95588.9	**1.8**	**2.6**

Table 6.2 Calculation of the standard error for DFLE by the Sullivan method using an abridged life table with institutionalized population

[1]	[1a]	[7]	[9]	*[10]*		[10a]	[13a]	*[15]*	[16a]	[17a]	[18a]	[19a]	[20a]
Age at start of interval	**Age group**	**Person years lived in age interval**	**Total Life Expectancy**	***Proportion of age group with disability***	***Proportion in institutions***	**Proportion of age group (total) with disability**	**Disability-free life expectancy (including inst)**	***Number in survey in age interval***				**Variance of DFLE′$_x$**	**Standard error of DFLE’$_x$**
x	**x – x+n**	**$_nL_x$**	**e_x**	π_x	I_x	π'_x	**DFLE′$_x$**	N_x	$S^2(\pi'_x)$	$L^2S^2(\pi'_x)$	$\Sigma L^2S^2(\pi'_x)$	$S^2(DFLE'_{x'})$	$S(DFLE'_x)$
0	0	99711.5	81.4	*0.000*	*0.000*	0.000	**65.7**	*52*	0.000000	0	1222587894	0.12226	**0.350**
1	1-4	398371.1	80.7	*0.048*	*0.000*	0.048	**64.9**	*230*	0.000199	31530146	1222587894	0.12315	**0.351**
5	5-9	497576.7	76.7	*0.030*	*0.000*	0.030	**61.2**	*257*	0.000113	28033671	1191057749	0.12019	**0.347**
10	10-14	497270.6	71.8	*0.072*	*0.000*	0.072	**56.4**	*275*	0.000243	60080466	1163024078	0.11751	**0.343**
15	15-19	496788.7	66.8	*0.098*	*0.000*	0.098	**51.8**	*281*	0.000315	77637184	1102943611	0.11158	**0.334**
20	20-24	496050.2	61.9	*0.087*	*0.001*	0.087	**47.3**	*342*	0.000232	57149854	1025306428	0.10400	**0.322**
25	25-29	495171.9	57.0	*0.096*	*0.001*	0.097	**42.8**	*402*	0.000217	53271216	968156574	0.09853	**0.314**
30	30-34	494136.5	52.1	*0.089*	*0.001*	0.090	**38.4**	*426*	0.000192	46807125	914885358	0.09346	**0.306**
35	35-39	492590.7	47.2	*0.142*	*0.001*	0.143	**33.9**	*437*	0.000280	67854606	868078233	0.08908	**0.298**
40	40-44	490097.0	42.4	*0.122*	*0.002*	0.124	**29.8**	*446*	0.000242	58167662	800223627	0.08278	**0.288**
45	45-49	486121.2	37.7	*0.195*	*0.002*	0.197	**25.6**	*438*	0.000359	84880535	742055966	0.07771	**0.279**
50	50-54	479425.4	33.0	*0.161*	*0.003*	0.164	**21.8**	*424*	0.000321	73703233	657175430	0.07023	**0.265**
55	55-59	470194.7	28.6	*0.298*	*0.003*	0.300	**18.0**	*407*	0.000513	113411822	583472197	0.06460	**0.254**
60	60-64	457774.7	24.1	*0.234*	*0.006*	0.239	**14.8**	*310*	0.000579	121336933	470060375	0.05430	**0.233**
65	65-69	439413.3	19.8	*0.257*	*0.009*	0.264	**11.4**	*398*	0.000479	92504259	348723442	0.04299	**0.207**
70	70-74	411279.0	15.7	*0.345*	*0.019*	0.357	**8.2**	*413*	0.000535	90527521	256219183	0.03489	**0.187**
75	75-79	361557.8	11.9	*0.431*	*0.047*	0.458	**5.6**	*310*	0.000727	95061913	165691662	0.02667	**0.163**
80	80-84	294456.7	8.7	*0.431*	*0.110*	0.494	**3.7**	*300*	0.000660	57222957	70629750	0.01631	**0.128**
85	85+	279205.1	5.4	*0.513*	*0.297*	0.658	**1.8**	*647*	0.000172	13406793	13406793	0.00496	**0.070**

Standard error of the Sullivan health expectancy as a proportion of life expectancy: Example 7

Often health expectancies at a particular age are expressed as proportions or percentages of life expectancies in order to illustrate what percentage of remaining life is spent in good health or disability-free. We have already calculated this quantity in Examples 1-4. In this example we show how to calculate the standard error of this quantity in two ways (as in Example 4) depending on whether or not the random variation of the mortality rates is taken into consideration. As in Example 4 we will use data for females in Belgium in 2004 and calculate the standard error of the percentage of remaining life spent disability-free. The first example will ignore the variation in the mortality rates (LE) and the second will include these.

Approximate standard errors ignoring the variance of the mortality rates

The calculations are shown in Table 7.1 and continue from Example 4 (shown in Table 4.1). The numbers at the top of the columns in Table 7.1 are the same as those in Table 4.1. For space we have omitted columns [6]-[7] but have reinstated column[14] which held the new quantity showing the percentage of remaining years spent disability-free (%$DFLE_x/e_x$) calculated as Column[14] = 100*column[13]/column[9].

1. The standard error of the DFLE was previously calculated and is shown in column[20]. The standard error of %$DFLE_x/e_x$ in column[27] is 100*column[20]/column[9].

2. If required, approximate 95% confidence intervals for %$DFLE_x/e_x$ are given by column[14] - 1.96*column[27] and column[14]+1.96*column[27]. As an example, in 2004 DFLE for women age 65 in Belgium was 12.3 years, 61.6% (95%CI 59.7% to 64.0%) of their life expectancy of 19.8 years.

Standard errors taking into account the variance of the mortality rates

Incorporating the uncertainty around the mortality rates is more complex and requires the derivation of an approximation to the variance of a ratio. This has been derived for a general ratio of random variables (Stuart and Ord 1998) but the solution for $DFLE_x/e_x$ is given in Appendix 1 paragraph 15[22]. The calculations will be shown in Table 7.2 which continues from Table 7.1 although for space we omit columns[11],[12],[15]-[25] but reinstating column[21].

In steps 1-7 below we calculate the variance of DLE, $S^2(DLE_x)$, as we shall need this - the formula for this was given in Appendix 1 paragraph 11[23]. We only need to calculate the first term in this formula, $S^2_{(2)}(DLE_x)$, which is the part of the variance resulting from the mortality rates, since we already have the second term, the part of the variance resulting from the prevalence in column[19] (see Example 4).

1. First we calculate DLE_x, which is e_x-$DFLE_x$, in column[28] as column[28]=column[9]-column[13].

2. For $S^2_{(2)}(DLE_x)$ we need to calculate $\ell_x^2[(1- a_x)n\pi_x + DLE_{x+n}]^2 S^2(p_x)$. From Example 4 we already have $S^2(p_x)$ in column[21] so we shall calculate $[(1- a_x)n\pi_x + DLE_{x+n}]$ in column[29].

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[22] See Appendix 1 paragraph 15. for the general formula.
[23] See Appendix 1 paragraph 11. for the general formula.

3. $[(1- a_x)n\pi_x + DLE_{x+n}]$ is calculated in column[29] by adding {(1-a_x)*n*column[10]} for an age group to the entry in column[28] for the next age group. Remember n is the length of the age interval and is therefore 1 for the first age group, 2 for the second and five for the remainder. For example for the 30-34 year age group we have a_{30} = 0.5, n = 5, π_x = 0.089 and DLE_{35} = 12.46, thus column[29]$_{35}$ = {(1-0.5)*5*0.089}+12.46 = 12.68.

4. Now we calculate $\ell_x^2[(1-a_x)n\pi_x + DLE_{x+n}]^2S^2(p_x)$ in column[30] as column[30]=column[29]*column[29]*column[6]*column[6]*column[21].

5. We now need to sum column[30] from age x to age 85+ years and the result is shown in column[31].

6. The part of the variance resulting from the mortality rates, $S^2_{(2)}(DLE_x)$, is shown in column[32] and equals column[31]/(column[6]*column[6]).

7. Column[33] contains the total variance, $S^2(DLE_x)$, which is the sum of column[19] and column[32].

8. Now in step 8 we calculate the standard error of DFLE as a proportion (percentage) of LE, $S(\%DFLE_x/e_x)$, in column[34] according to the formula in Appendix 1 paragraph 15. For this we need 6 quantities: e_x (column[9]), $DFLE_x$ (column[13]), DLE_x (column[28]), and their variances which are found in column[21], column[26] and column[33] respectively. Thus column[34]=100*(column[9]*column[28]*column[26]-column[13]*column[28]* column[21] +column[13]*column[9]*column[33])/(column[9]*column[9]).

9. It is worth noting that in this example with the Belgian data there is very little difference between the standard error ignoring the mortality variation (in column[27]) and that taking the mortality variation into account (in column[34]).

10. If required, approximate 95% confidence intervals for $\%DFLE_x/e_x$ are given by column[14] - 1.96*column[34] and column[14]+1.96*column[34].

Table 7.1 Calculation of the standard error for %DFLE/TLE by the Sullivan method using an abridged life table, ignoring the variance from the mortality rates

[1]	[1a]	[9]	*[10]*	[11]	[12]	[13]	[14]	*[15]*	[16]	[17]	[18]	[19]	[20]	[27]
Age at start of interval	**Age group**	**Total Life Expectancy**	***Proportion of age group with disability***	**Person years lived without disability in interval**	**Total years lived without disability from age x**	**Disability-free life expectancy**	**Proportion of remaining life spent disability-free**	***Number in survey in age interval***				**Variance of $DFLE_x$**	**Standard error of $DFLE_x$**	**Standard error of $\%DFLE_x / e_x$**
x	**x – x+n**	**e_x**	**π_x**	**$(1-\pi_x)*L_x$**	**$\Sigma[(1-\pi_x)*L_x]$**	**$DFLE_x$**	**$\%DFLE_x / e_x$**	***N_x***	**$S^2(\pi_x)$**	**$L^2S^2(\pi_x)$**	**$\Sigma L^2S^2(\pi_x)$**	**$S^2(DFLE_x)$**	**$S(DFLE_x)$**	**$S(\%DFLE_x/e_x)$**
0	0	81.4	*0.000*	99711.5	6654230.9	66.5	**81.8**	*52*	0.000000	0	1261478651	0.12615	0.355	**0.436**
1	1-4	80.7	*0.048*	379249.3	6554519.4	65.8	**81.5**	*230*	0.000197	31530146	1261478651	0.12706	0.356	**0.442**
5	5-9	76.7	*0.030*	482649.4	6175270.1	62.0	**80.8**	*257*	0.000112	28033671	1229948505	0.12412	0.352	**0.459**
10	10-14	71.8	*0.072*	461467.1	5692620.7	57.2	**79.7**	*275*	0.000243	60080466	1201914834	0.12144	0.348	**0.485**
15	15-19	66.8	*0.098*	448103.4	5231153.6	52.6	**78.7**	*281*	0.000314	77637184	1141834368	0.11551	0.340	**0.509**
20	20-24	61.9	*0.087*	452893.8	4783050.2	48.2	**77.8**	*342*	0.000231	57149854	1064197184	0.10794	0.329	**0.531**
25	25-29	57.0	*0.096*	447635.4	4330156.3	43.7	**76.6**	*402*	0.000215	52932898	1007047330	0.10248	0.320	**0.562**
30	30-34	52.1	*0.089*	450158.4	3882520.9	39.2	**75.3**	*426*	0.000190	46472141	954114432	0.09747	0.312	**0.599**
35	35-39	47.2	*0.142*	422642.8	3432362.5	34.8	**73.6**	*437*	0.000279	67649820	907642291	0.09314	0.305	**0.646**
40	40-44	42.4	*0.122*	430305.2	3009719.7	30.6	**72.2**	*446*	0.000241	57687743	839992471	0.08689	0.295	**0.695**
45	45-49	37.7	*0.195*	391327.6	2579414.6	26.4	**70.1**	*438*	0.000358	84692606	782304728	0.08193	0.286	**0.760**
50	50-54	33.0	*0.161*	402237.9	2188087.0	22.6	**68.5**	*424*	0.000318	73225784	697612123	0.07455	0.273	**0.827**
55	55-59	28.6	*0.298*	330076.7	1785849.1	18.8	**65.8**	*407*	0.000514	113635608	624386339	0.06913	0.263	**0.921**
60	60-64	24.1	*0.234*	350655.4	1455772.4	15.6	**64.9**	*310*	0.000578	121167615	510750731	0.05900	0.243	**1.007**
65	65-69	19.8	*0.257*	326484.0	1105117.0	12.3	**61.9**	*398*	0.000480	92637146	389583116	0.04802	0.219	**1.105**
70	70-74	15.7	*0.345*	269387.8	778632.9	9.1	**57.8**	*413*	0.000547	92551503	296945970	0.04044	0.201	**1.280**
75	75-79	11.9	*0.431*	205726.4	509245.2	6.5	**54.5**	*310*	0.000791	103414969	204394467	0.03290	0.181	**1.529**
80	80-84	8.7	*0.431*	167545.9	303518.8	4.6	**52.9**	*300*	0.000817	70877957	100979498	0.02332	0.153	**1.752**
85	85+	5.4	*0.513*	135972.9	135972.9	2.6	**48.7**	*647*	0.000386	30101542	30101542	0.01114	0.106	**1.965**

Table 7.2 Calculation of the standard error for %DFLE/TLE by the Sullivan method using an abridged life table, taking into account the variance from the mortality rates

[1]	[1a]	[9]	*[10]*	[13]	[14]	[21]	[26]	[27]	[28]	[29]	[30]	[31]	[32]	[33]	[34]
Age at start of interval	Age group	Total Life Expectancy	*Proportion of age group with disability*	Disability-free life expectancy	Proportion of remaining life spent disability-free		Total Variance of DFLE	Standard error of $\%DFLE_x$ $/e_x$ (ignoring mortality variation)	Years with disability				Variance of DLE (due to mortality)	Total Variance of DLE	Standard error of $\%DFLE_x$ $/e_x$ (including mortality variation)
x	x – x+n	e_x	π_x	$DFLE_x$	$\%DFLE_x/e_x$	$S^2(q_x)$	$S^2(DFLE_x)$	$S(\%DFLE_x/e_x)$	DLEx				$S^2_{(2)}(DLE)$	$S^2(DLE)$	$S(\%DFLE_x/e_x)$
0	0	81.4	*0.000*	66.5	**81.8**	0.00000006	0.12749	**0.436**	14.83	14.88	142099.08	3053296.11	0.00031	0.12645	**0.437**
1	1-4	80.7	*0.048*	65.8	**81.5**	0.00000002	0.12813	**0.442**	14.88	14.80	36563.35	2911197.02	0.00029	0.12736	**0.443**
5	5-9	76.7	*0.030*	62.0	**80.8**	0.00000001	0.12512	**0.459**	14.71	14.64	22613.81	2874633.67	0.00029	0.12441	**0.460**
10	10-14	71.8	*0.072*	57.2	**79.7**	0.00000001	0.12241	**0.485**	14.56	14.39	20144.48	2852019.86	0.00029	0.12173	**0.486**
15	15-19	66.8	*0.098*	52.6	**78.7**	0.00000002	0.11645	**0.509**	14.21	13.99	43041.47	2831875.38	0.00029	0.11580	**0.510**
20	20-24	61.9	*0.087*	48.2	**77.8**	0.00000003	0.10883	**0.531**	13.74	13.55	47090.49	2788833.91	0.00028	0.10823	**0.532**
25	25-29	57.0	*0.096*	43.7	**76.6**	0.00000003	0.10331	**0.562**	13.33	13.11	50024.38	2741743.42	0.00028	0.10276	**0.563**
30	30-34	52.1	*0.089*	39.2	**75.3**	0.00000003	0.09825	**0.599**	12.87	12.68	50130.38	2691719.04	0.00027	0.09774	**0.600**
35	35-39	47.2	*0.142*	34.8	**73.6**	0.00000005	0.09389	**0.646**	12.46	12.15	72173.99	2641588.65	0.00027	0.09342	**0.648**
40	40-44	42.4	*0.122*	30.6	**72.2**	0.00000008	0.08758	**0.695**	11.80	11.56	97635.27	2569414.67	0.00027	0.08716	**0.697**
45	45-49	37.7	*0.195*	26.4	**70.1**	0.00000013	0.08257	**0.760**	11.26	10.88	148720.25	2471779.40	0.00026	0.08219	**0.762**
50	50-54	33.0	*0.161*	22.6	**68.5**	0.00000024	0.07513	**0.827**	10.39	10.17	230462.33	2323059.15	0.00025	0.07480	**0.829**
55	55-59	28.6	*0.298*	18.8	**65.8**	0.00000033	0.06961	**0.921**	9.76	9.21	250936.65	2092596.82	0.00023	0.06936	**0.923**
60	60-64	24.1	*0.234*	15.6	**64.9**	0.00000061	0.05940	**1.007**	8.47	8.14	351571.01	1841660.18	0.00021	0.05922	**1.010**
65	65-69	19.8	*0.257*	12.3	**61.9**	0.00000086	0.04832	**1.105**	7.56	7.27	370149.21	1490089.16	0.00018	0.04821	**1.108**
70	70-74	15.7	*0.345*	9.1	**57.8**	0.00000135	0.04065	**1.280**	6.63	6.27	389321.38	1119939.95	0.00015	0.04059	**1.283**
75	75-79	11.9	*0.431*	6.5	**54.5**	0.00000260	0.03305	**1.529**	5.40	5.18	434158.55	730618.57	0.00012	0.03302	**1.532**
80	80-84	8.7	*0.431*	4.6	**52.9**	0.00000466	0.02339	**1.752**	4.11	3.83	296460.02	296460.02	0.00007	0.02339	**1.754**
85	85+	5.4	*0.513*	2.6	**48.7**			**1.965**	2.76						**1.965**

Appendix 1

Here we give further technical details of the calculation of health expectancy by the Sullivan method. Since these calculations rely on quantities derived from life tables, further explanations are available elsewhere(Chiang 1985).

Sullivan health expectancy with a complete life table

Column numbers refer to those in Tables 1-3.

1. Let the age intervals (column [1]) for the calculations be $[x, x+1)$. Let ω be the last interval $[\omega, \infty)$ and $x=0,1,..., \omega$.

2. Column[4] gives the age-specific death rates $m_x = D_x/(P_x .T)$ where D_x is the number of deaths at age x registered during the period of length T and P_x is the average population exposed to risk (mid-year population). Almost all countries are able to give deaths by both age definitions: the age reached at the last birthday (age at the observation date) and the age that is, will or would be reached at the birthday in a given calendar year (the age on 31 December). Here, D_x is based on the age at last birthday. In the example T is one calendar year and P_x is the midyear population for the calendar year (or average of the calendar years) corresponding to the survey in which the prevalence of the health state is estimated.

3. Column[5] gives q_x, the proportion of those alive at age x who die in the interval $[x,x+1)$. These are derived from the corresponding age-specific deaths rates of the current population, using the formula:

$$q_x = \frac{m_x}{1+(1-a_x)m_x} \qquad x=0,1,..., \omega.$$

   Each of those people who die in the interval $[x, x+1)$ have lived x complete years plus a fraction, a_x, of the last year. a_x is often considered to be 0.5 on average, assuming that deaths are uniformly distributed in the interval, except for the first four years of life (ages 1 to 4 years). However, the result is close enough to reality to allow the approximation for these age groups also, certainly for single-year age groups.

4. Eurostat (Calot and Sardon 2003) proposes another formula for the calculation of q_x before the age of 1 year:

$$q_0 = 1-(1-\frac{D_0^1}{P_0^1})*(1-\frac{D_0^2}{B}) \text{ with } D_0 = D_0^1 + D_0^2$$

   The number of deaths between age 0 and age 1 (D_0), occurring during the year of interest, can be decomposed into the number of deaths occurring between the first of January of this year and the first birthday (D_0^1) and the number of deaths occurring between birth and the end of the year (D_0^2). P_0^1 is the population at age 0 by January 1st of the calendar year and B is the number of births of the year.

5. We begin the calculation of the number surviving to age x, ℓ_x (column[6]), by fixing this at 100,000 for the first age in the table. For the remaining entries in column[6] we use the equation $\ell_{x+1} = \ell_x(1-q_x)$ for $x=0,1,..., \omega-1$.

6. The total number of years lived by the cohort in the interval $[x, x+1)$ is given by L_x (column[7]). Apart from the last age interval ω (see 8. below), the members of the cohort who survive the interval (ℓ_{x+1}) will each contribute one year (since the age intervals are one year). The remainder (ℓ_x - ℓ_{x+1}) will each contribute a fraction a_x. Hence for $x=0,1,..., \omega-1$

$$\begin{aligned} L_x &= \ell_{x+1} + a_x(\ell_x - \ell_{x+1}) \\ &= (1-a_x)\ell_{x+1} + a_x\ell_x \\ &= \frac{(\ell_{x+1} + \ell_x)}{2} \text{ if } \mathrm{a_x} = 0.5 \end{aligned}$$

7. Before the age of 1, Eurostat uses another formula, which assumes that 80% of the deaths happen in the first months of life:

$$L_x = 0.2 * \ell_x + 0.8 * \ell_{x+1} \text{ with } \ell_x = \ell_0 = 1$$

8. Often life tables are made up to age 100 years as little information is available after this age. For the last age interval ω we use the formula:

$$L_\omega = \frac{\ell_\omega}{m_\omega}$$

where M_ω is the death rate for the last age interval.

Quantification of mortality above the age of 80 years and evaluation of survival at these ages raise substantial problems. A number of methodological and practical difficulties are involved. Errors in data play a much more serious role in the case of the elderly than in other groups of the population and errors are present in both the death and population statistics (Kannisto 1988). However, the biases are generally small compared to the standard errors and can safely be ignored in most cases.

9. The life expectancy at age x, e_x (column[9]), is given by

$$e_x = \frac{1}{\ell_x}\sum_{i=x}^{\omega} L_i$$

The calculation of health expectancy follows similar lines. If we assume two states called Disability-free (*DF*) and with disability (*D*) then the Disability-Free Life Expectancy at age *x* ($DFLE_x$) and the life expectancy with disability (DLE_x) are defined by:

$$DFLE_x = \frac{1}{\ell_x}\sum_{i=x}^{\omega} L_i(DF) \text{ and}$$

$$DLE_x = \frac{1}{\ell_x}\sum_{i=x}^{\omega} L_i(D)$$

where $L_i(DF)$ and $L_i(D)$ are the number of person years lived from age x onwards in the states DF (without disability) and D (with disability) respectively. Using the Sullivan method as an approximation of health expectancy (multistate) leads to the hypothesis that

$$L_i(D) = \pi_i L_i \qquad i=0,...,\omega,$$

where π_i is the prevalence of disability at age I. Thus for $x=0,...,\omega$

$$DFLE_x = \frac{1}{\ell_x}\sum_{i=x}^{\omega}(1-\pi_i)L_i \text{ and}$$

$$DLE_x = \frac{1}{\ell_x}\sum_{i=x}^{\omega}\pi_i L_i \quad .$$

Amendments for an abridged life table

10. Assume the length of the age interval is no longer one year but n (this may vary for different age groups but the subscript is omitted for clarity). Then for the age interval [x, $x+n$):

$$_n m_x = \frac{_n D_x}{_n P_x \times T} \qquad x=0,1,...,\omega,$$

$$q_x = \frac{n\,_n m_x}{1+n(1-a_x)\,_n m_x} \qquad x=0,1...,\omega.$$

and

$$\begin{aligned} _n L_x &= n\ell_{x+1} + na_x(\ell_x - \ell_{x+1}) \\ &= n(1-a_x)\ell_{x+1} + na_x\ell_x \end{aligned}$$

Calculation of variance of Sullivan health expectancy

The same notation is taken as above and the general formulae shown for use with an abridged life table with ages x, $x+n$,... and $x=0,...,\omega$.

11. Mathers (1991) shows that the variances of $DFLE_x$ and DLE_x can be approximated by:

$$S^2(DLE_x) = \frac{1}{\ell_x^2}\sum_{x=0}^{\omega-1}\ell_x^2[(1-a_x)n\pi_x + DLE_{x+n}]^2 S^2(p_x) + \frac{1}{\ell_x^2}\sum_{x=0}^{\omega}L_x^2 S^2(\pi_x)$$

$$S^2(DFLE_x) = \frac{1}{\ell_x^2}\sum_{x=0}^{\omega-1}\ell_x^2[(1-a_x)n(1-\pi_x) + DFLE_{x+n}]^2 S^2(p_x) + \frac{1}{\ell_x^2}\sum_{x=0}^{\omega}L_x^2 S^2(1-\pi_x)$$

where $p_x = \ell_{x+n}/\ell_x$ and a_x, as before, is the fraction of the interval lived by those who die in the interval.

If the sample size of the survey producing the prevalence ratios is not very large compared to the population on which the mortality data are based then $S^2(p_x)$ is negligible and the first terms of the above equations can be ignored (Newman 1988).

In general $S^2(\pi_x) = S^2(1-\pi_x)$ and both can be approximated by

$$S^2(\pi_x) = \frac{\pi_x(1-\pi_x)}{N_x}$$

where N_x is the number of persons in the age interval $[x, x+n)$ participating in the prevalence survey. However it may be necessary to include this term if the survey has a complex sampling design (e.g. stratified or cluster). A simple approximation in the case of complex sampling design is to use the weighted π_x and the unweighted N_x.

Hence the variances of $DFLE_x$ and DLE_x can be approximated by

$$S^2(DLE_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{\omega} L_x^2 S^2(\pi_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{\omega} L_x^2 \frac{\pi_x(1-\pi_x)}{N_x}$$

and

$$S^2(DFLE_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{\omega} L_x^2 S^2(1-\pi_x) \approx \frac{1}{\ell_x^2}\sum_{x=0}^{\omega} L_x^2 \frac{\pi_x(1-\pi_x)}{N_x}$$

Testing the equality of two Sullivan health expectancies

12. Since the estimates of health expectancy are the means of random variables assumed to be independent, application of the central limit theorem means that they can be assumed to have normal distributions. Therefore the hypothesis of equality of two health expectancies, say $DFLE_{(1)}$ and $DFLE_{(2)}$, may be tested by the following Z-score:

$$Z = \frac{DFLE_{(1)} - DFLE_{(2)}}{\sqrt{S^2(DFLE_{(1)} - DFLE_{(2)})}}$$

If we denote by $S(DFLE_{(1)})$ and $S(DFLE_{(2)})$ the standard errors of $DFLE_{(1)}$ and $DFLE_{(2)}$ respectively then since

$$S^2(DFLE_{(1)} - DFLE_{(2)}) \le [S(DFLE_{(1)}) + S(DFLE_{(2)})]^2$$

then we compute the approximate Z-score (a conservative approximation) as

$$Z = \frac{DFLE_{(1)} - DFLE_{(2)}}{S(DFLE_{(1)}) + S(DFLE_{(2)})}$$

The hypothesis of equality is rejected if the absolute value of the Z-score is too large, say ≥ 1.96 (see Appendix 2 for critical values of the Z-score).

Incorporating data from separate surveys of community-living and institutionalized

13. We denote by π_x the proportion with disability estimated from the survey and I_x the proportion in institutions in the age interval x to $x+n$. Then π'_x the proportion disabled or in institutions is given by

$$\pi'_x = (1- I_x)\ \pi_x + I_x\ .$$

14. Since I_x is derived from Census data we do not consider it to be a random variable. Therefore $S^2(\pi'_x)$ can be approximated by

$$S^2(\pi_x) = (1 - I_x)^2 \frac{\pi'_x(1-\pi'_x)}{N_x}$$

Calculation of variance of DFLE as a proportion of LE (%DFLE/LE)

If R and S are random variables then Kendall derives an approximation to the variance of R/S, var(R/S), using Taylor expansions. Thus

$$Var(R/S) \approx \frac{E^2R}{E^2S}\left[\frac{Var(R)}{E^2R} - \frac{2Cov(R,S)}{ER.ES} + \frac{Var(S)}{E^2S}\right]$$

Replacing R by DFLE and S by LE gives

$$Var(DFLE/LE) \approx \frac{DFLE^2}{LE^2}\left[\frac{Var(DFLE)}{DFLE^2} - \frac{2Cov(DFLE,LE)}{DFLE.LE} + \frac{Var(LE)}{LE^2}\right]$$

Since DLE=LE-DFLE then

$$Var(DLE) = Var(LE - DFLE) = Var(LE) + Var(DFLE) - 2Cov(DFLE,LE)$$

$$\therefore -2Cov(DFLE,LE) = Var(DLE) - Var(LE) - Var(DFLE)$$

Substituting for -2Cov(DFLE,LE) in the earlier equation gives

$$Var(DFLE/LE) \approx \frac{DFLE^2}{LE^2}\left[\frac{Var(DFLE)}{DFLE^2} - \frac{[Var(DLE) - Var(LE) - Var(DFLE)]}{DFLE.LE} + \frac{\mathrm{var}(LE)}{LE^2}\right]$$

$$= \frac{DFLE^2}{LE^2}\left[\frac{(LE - DFLE)Var(DFLE)}{DFLE^2.LE} + \frac{Var(DLE)}{DFLE.LE} - \frac{(LE-DFLE)Var(LE)}{DFLE.LE^2}\right]$$

$$= \frac{DFLE}{LE^2}\left[\frac{DLE.Var(DFLE)}{DFLE.LE} + \frac{Var(DLE)}{LE} - \frac{DLE.Var(LE)}{LE^2}\right]$$

$$= \frac{1}{LE^4}\left[LE.DLE.Var(DFLE) + LE.DFLE.Var(DLE) - DFLE.DLE.Var(LE)\right]$$

We know all these quantities and reverting to the notation and values in paragraph 9 we have

$$Var(DFLE_x/e_x) = \frac{1}{e_x^{\,4}}\left[e_x.DLE_x.S^2(DFLE_x) + e_x.DFLE_x.S^2(DLE_x) - DFLE_x.DLE_x.S^2(p_x)\right]$$

It is easily seen that if the variance of the mortality rates can be ignored then $S^2(p_x)$=0 and $S^2(DFLE_x)$= $S^2(DLE_x)$ and $Var(DFLE_x/e_x) = \frac{1}{e_x^{\,2}} S^2(DFLE_x)$

Remember, the $Var(\%DFLE_x/e_x) = 100^2 Var(DFLE_x/e_x)$ and $SE(\%DFLE_x/e_x) = 100\sqrt{Var(DFLE_x/e_x)}$.

Appendix 2

Table A1. Critical values of the z-statistic

z critical values	**Level of significance for a *two*-tailed test**	**Level of significance for a *one*-tailed test**
1.28	0.20	0.10
1.645	0.10	0.05
1.96	0.05	0.025
2.33	0.02	0.01
2.58	0.01	0.005
3.09	0.002	0.001
3.29	0.001	0.0005

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